1. 9709/32/F/M/18 Q5

The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = 1 - 2\cos 2t,$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 2\tan t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient of the normal is 2. Give your answer correct to 3 significant figures. [2]

2. 9709/31/M/J/18 Q3

A curve has equation $y = \dfrac{e^{3x}}{\tan \frac{1}{2}x}$. Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

3. 9709/32/M/J/18 Q5

The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

4. 9709/33/M/J/18 Q8

The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x^2 - y^2}{y^2 + 2xy}$. [4]

(ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

5. 9709/31/O/N/18 Q4

The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

6. 9709/32/O/N/18 Q7

A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

7. 9709/32/O/N/18 Q5

The equation of a curve is $y = x\ln(8 - x)$. The gradient of the curve is equal to 1 at only one point, when $x = a$.

(i) Show that a satisfies the equation $x = 8 - \dfrac{8}{\ln(8 - x)}$. [3]

(ii) Verify by calculation that a lies between 2.9 and 3.1. [2]

8. 9709/33/O/N/18 Q4

The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in terms of θ. [3]

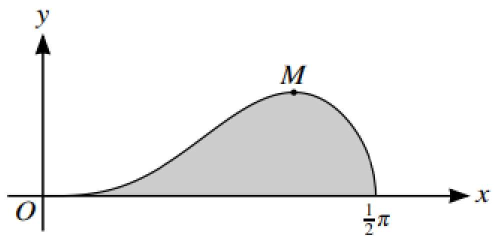
(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

9. **9709/32/F/M/19 Q5**

The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that

$$\frac{dy}{dx} = \frac{1}{\cos x \sqrt{(\cos 2x)}}.$$ [5]

10. 9709/32/F/M/19 Q10



The diagram shows the curve $y = \sin^3 x \sqrt{(\cos x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Showing all your working, find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

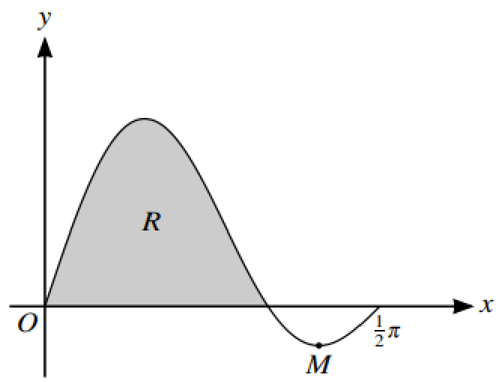
11. 9709/31/M/J/19 Q3

Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates (1, 3). [4]

12. 9709/32/M/J/19 Q4

Find the exact coordinates of the point on the curve $y = \dfrac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

13. 9709/32/M/J/19 Q10



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

14. 9709/33/M/J/19 Q4

The equation of a curve is $y = \dfrac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

(i) Show that $\dfrac{dy}{dx}$ is always negative. [3]

(ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^{a} + 1 = 0$. Hence find the exact value of a. [4]

15. 9709/33/M/J/19 Q7

The curve $y = \sin\left(x + \frac{1}{3}\pi\right)\cos x$ has two stationary points in the interval $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) By considering the formula for $\cos(A + B)$, show that, at the stationary points on the curve, $\cos\left(2x + \frac{1}{3}\pi\right) = 0$. [2]

(iii) Hence find the exact x-coordinates of the stationary points. [3]

16. 9709/33/M/J/19 Q3

The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\frac{dy}{dx} = \operatorname{cosec} 2t$. [5]

17. 9709/32/O/N/19 Q2

The curve with equation $y = \dfrac{e^{-2x}}{1 - x^2}$ has a stationary point in the interval $-1 < x < 1$. Find $\dfrac{dy}{dx}$ and hence find the x-coordinate of this stationary point, giving the answer correct to 3 decimal places.

[5]

18. 9709/32/O/N/19 Q5

The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

19. 9709/33/O/N/19 Q4

(i) By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3\cot x$ can be written in the form $\tan^4 x - 12\tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

20. 9709/32/F/M/20 Q7

The equation of a curve is $x^3 + 3xy^2 - y^3 = 5$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + y^2}{y^2 - 2xy}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the y-axis. [5]

21. 9709/31/M/J/20 Q4

The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

22. 9709/32/M/J/20 Q4

A curve has equation $y = \cos x \sin 2x$.

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

23. 9709/33/M/J/20 Q4

The equation of a curve is $y = x\tan^{-1}\left(\frac{1}{2}x\right)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) The tangent to the curve at the point where $x = 2$ meets the y-axis at the point with coordinates $(0, p)$.

Find p. [3]

24. 9709/31/O/N/20 Q3

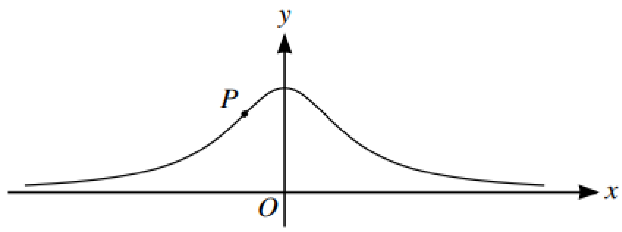
The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$. [5]

25. 9709/32/O/N/20 Q5



The diagram shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

26. 9709/33/O/N/20 Q3

The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$. [5]

27. 9709/31/M/J/21 Q6

The parametric equations of a curve are

$$x = \ln(2 + 3t), \qquad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive. [5]

(b) Find the equation of the tangent to the curve at the point where it intersects the y-axis. [3]

28. 9709/32/M/J/21 Q3

The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

29. 9709/33/M/J/21 Q3

The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)e^{-2t},$$

where $t > -2$.

(a) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [5]

(b) Find the exact y-coordinate of the stationary point of the curve. [2]

30. 9709/31/O/N/21 Q3

The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

31. 9709/32/O/N/21 Q9

The equation of a curve is $ye^{2x} - y^2e^x = 2$.

(a) Show that $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$. [4]

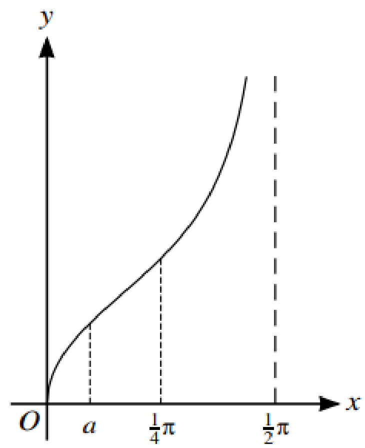
(b) Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

32. 9709/32/O/N/21 Q11

The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leqslant x < \frac{1}{2}\pi$.

(a) Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



(b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

33. 9709/33/O/N/21 Q7

The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x + y - 1}{2(x + y) + 1}$. [4]

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

34. 9709/32/F/M/22 Q4

The parametric equations of a curve are

$$x = 1 - \cos\theta, \qquad y = \cos\theta - \tfrac{1}{4}\cos 2\theta.$$

Show that $\dfrac{dy}{dx} = -2\sin^2\left(\tfrac{1}{2}\theta\right)$. [5]

35. 9709/31/M/J/22 Q8

The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y. [4]

The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle α.

(b) Find the exact value of $\tan \alpha$. [5]

36. 9709/32/M/J/22 Q4

The equation of a curve is $y = \cos^3 x\sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

37. 9709/32/M/J/22 Q7

The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

38. 9709/33/M/J/22 Q4

The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b\sin 2x)e^{-4x}$, where a and b are constants. [4]

(b) Hence find the exact x-coordinates of the two stationary points. [3]

39. 9709/33/M/J/22 Q6

The parametric equations of a curve are $x = \dfrac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{\cos t}{\sin^2 t}$. [5]

(b) Find the equation of the tangent to the curve at the point where $y = 0$. [3]

40. 9709/31/O/N/22 Q7

The equation of a curve is $y = \dfrac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1}\left(\sqrt[3]{\dfrac{\cos a + 2a \sin a}{12}}\right)$. [3]

(b) Verify by calculation that a lies between 0.9 and 1. [2]

41. 9709/33/O/N/22 Q4

The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot t$. [5]

42. 9709/33/O/N/22 Q8

The curve with equation $y = \dfrac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$. [3]

(b) Verify by calculation that p lies between 2.5 and 3. [2]

43. 9709/32/F/M/23 Q5

The parametric equations of a curve are

$$x = te^{2t}, \qquad y = t^2 + t + 3.$$

(a) Show that $\frac{dy}{dx} = e^{-2t}$. [3]

(b) Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0,\ 3 - \frac{1}{e^4}\right)$. [3]

44. 9709/31/M/J/23 Q5

The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

45. 9709/32/M/J/23 Q7

The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x + 2y}{2x + 3y}$. [4]

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

46. 9709/33/M/J/23 Q4

The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\frac{dy}{dx} = (2 - \sin\theta)^2$. [5]

47. 9709/31/O/N/23 Q6

The parametric equations of a curve are

$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for $t > 0$.

(a) Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t. [3]

(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2. [3]

48. 9709/32/O/N/23 Q2

The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer. [4]

49. 9709/33/O/N/23 Q5

Find the exact coordinates of the stationary points of the curve $y = \dfrac{e^{3x^2-1}}{1-x^2}$. [6]

50. 9709/32/F/M/24 Q6

The equation of a curve is $2y^2 + 3xy + x = x^2$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x - 3y - 1}{4v + 3x}$. [4]

(b) Hence show that the curve does **not** have a tangent that is parallel to the x-axis. [3]