1. 9709/32/F/M/18 Q5

Question	Answer	Marks
5(i)	State correct derivative of x or y with respect to t	B1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1
	Obtain $\frac{dy}{dx} = \frac{4\sin 2t}{2 + 2\cos 2t}$, or equivalent	A1
	Use double angle formulae throughout	M1
	Obtain the given answer correctly **AG**	A1
		5
5(ii)	State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$	B1
	Obtain answer $x = -0.961$	B1
		2

2. 9709/31/M/J/18 Q3

Question	Answer	Marks
3	Use quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a \sin x = b$	M1 *
	Solve for x	M1(dep*)
	Obtain answer 0.340	A1
	Obtain second answer 2.802 and no other in the given interval	A1
		6

3. 9709/32/M/J/18 Q5

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	B1	$3x^2+6xy+3x^2\frac{dy}{dx}-3y^2\frac{dy}{dx}=0$
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	Given answer so check working carefully
	Obtain the given answer	A1	
		4	
5(ii)	Equate derivative to – 1 and solve for y	M1*	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	M1(dep*)	
	Obtain answer $(1, -2)$	A1	
	Obtain answer $(\sqrt[3]{3}, 0)$	B1	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

4. 9709/33/M/J/18 Q8

Question	Answer	Marks
8(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1
	State or imply $3y^2+6xy\frac{dy}{dx}$ as derivative of $3xy^2$	B1
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
	Total:	4
8(ii)	Equate denominator to zero and solve for y	M1*
	Obtain $y = 0$ and $x = a$	A1
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	M1(dep*)
	Obtain $x = -a$	A1
	Obtain $y = 2a$	A1
	Total:	5

5. 9709/31/O/N/18 Q4

Question	Answer	Marks
4(i)	Obtain $\frac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	B1
	Use $dy/dx = dy/d\theta \div dx/d\theta$	M1
	Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	A1
		3
4(ii)	Equate denominator to zero and use any correct double angle formula	M1*
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	A1
	Solve for θ	depM1*
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	A1
		4

6. 9709/31/O/N/18 Q7

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Question	Answer	Marks	Guidance
7(i)	Use correct quotient or product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{-3\sin x(2+\sin x) - 3\cos x\cos x}{(2+\sin x)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	M1	
	Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$	M1	$-6\sin x - 3 = 0 \Rightarrow \sin x = \dots$
	Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW	A1 + A1	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	

7. 9709/32/O/N/18 Q5

Question	Answer	Marks	Guidance
5(i)	Use product rule on a correct expression	M1	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	A1	$\frac{dy}{dx}=\ln(8-x)-\frac{x}{8-x}$
	Equate derivative to 1 and obtain $x=8-\frac{8}{\ln(8-x)}$	A1	**Given answer:** check carefully that it follows from correct working
			Condone the use of a for x throughout
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.9$ and $x = 3.1$	M1	$8-\frac{8}{\ln 5.1}=3.09>2.9$, $8-\frac{8}{\ln 4.9}=2.97<3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	A1	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

8. 9709/33/O/N/18 Q4

Question	Answer	Marks
4(i)	Obtain $\frac{dx}{d\theta}=2\cos\theta+2\cos 2\theta$ or $\frac{dy}{d\theta}=-2\sin\theta-2\sin 2\theta$	B1
	Use $dy/dx = dy/d\theta \div dx/d\theta$	M1
	Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin\theta+2\sin 2\theta}{2\cos\theta+2\cos 2\theta}$	A1
		3
4(ii)	Equate denominator to zero and use any correct double angle formula	M1*
	Obtain correct 3-term quadratic in cos θ in any form	A1
	Solve for θ	depM1*
	Obtain $x=3\sqrt{3}/2$ and $y=\frac{1}{2}$, or exact equivalents	A1
		4

9. 9709/32/F/M/19 Q5

Question	Answer	Marks
5	State $\cos y \frac{dy}{dx}$ as derivative of $\sin y$	B1
	State correct derivative in terms of x and y, e.g. $\sec^2 x / \cos y$	B1
	State correct derivative in terms of x, e.g. $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$	B1
	Use double angle formula	M1
	Obtain the given answer correctly	A1
		5

10. 9709/32/F/M/19 Q10

Question	Answer	Marks	Guidance
10(i)	State or imply $du = -\sin x\, dx$	B1	
	Using Pythagoras express the integral in terms of u	M1	
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	A1	
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{7}u^{\frac{7}{2}}$, or equivalent	A1	
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}} + bu^{\frac{7}{2}}$	M1	Or substitute original limits correctly in an integral of the form $a(\cos x)^{\frac{3}{2}} + b(\cos x)^{\frac{7}{2}}$
	Obtain answer $\frac{8}{21}$	A1	
		6	
10(ii)	Use product rule and chain rule at least once	M1	
	Obtain correct derivative in any form	A1 + A1	
	Equate derivative to zero and obtain a horizontal equation in integral powers of $\sin x$ and $\cos x$	M1	
	Use correct methods to obtain an equation in one trig function	M1	
	Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183	A1	
		6	

11. 9709/31/M/J/19 Q3

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Question	Answer	Marks	Guidance
3	State or imply $3y^2 + 6xy\frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1	$\left(\frac{dy}{dx} = \frac{x^2+y^2}{y^2-2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	A1	3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen
		4	

12. 9709/32/M/J/19 Q4

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Question	Answer	Marks	Guidance
4	Use correct quotient rule	M1	Allow use of correct product rule on $x\times(1+\ln x)^{-1}$
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{(1+\ln x) - x\times\frac{1}{x}}{(1+\ln x)^2} = \left(\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in ln x or (1+ ln x)	M1	Horizontal form. Accept $\ln x = \frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2 - 2\ln x + 1 = 0$	A1	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in ln x for x	M1	Must see working if solving incorrect quadratic
	Obtain answer $x = \mathrm{e}$	A1	Accept e^1
	Obtain answer $y = \frac{1}{2}\mathrm{e}$	A1	Exact only with no decimals seen before the exact value. Accept $\frac{\mathrm{e}^1}{2}$ but not $\frac{\mathrm{e}}{1+\ln \mathrm{e}}$
		7	

13. 9709/32/M/J/19 Q10

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Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x + \sin 2x)$	M1	
	Obtain $\sin 3x\cos x = \frac{1}{2}(\sin 4x + \sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x = 0$ and $x = \frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	

Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \frac{-1 \pm \sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = ...$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer $x = 1.29$ only	A1	
		5	

14. 9709/33/M/J/19 Q4

Question	Answer	Marks
4(i)	Use the quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Reduce to $-\frac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	A1
		3

Question	Answer	Marks
4(ii)	Equate derivative to – 1 and obtain the given equation	B1
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	B1
	Solve for a	M1
	Obtain answer $a = \ln\left(2 + \sqrt{3}\right)$ and no other	A1
		4

15. 9709/33/M/J/19 Q7

Question	Answer	Marks
7(i)	Use product rule	M1
	Obtain correct derivative in any form	A1
		2
7(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	M1
	Obtain the given equation	A1
		2
7(iii)	Use correct method to solve for x	M1
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	A1
		3

16. 9709/31/O/N/19 Q3

Question	Answer	Marks	Guidance
3	State $\frac{dx}{dt} = 2 + 2\cos 2t$	B1	
	Use the chain rule to find the derivative of y	M1	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1 - \cos 2t}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \operatorname{cosec} 2t$ correctly	A1	AG
		5	

17. 9709/32/O/N/19 Q2

Question	Answer	Marks	Guidance
2	Use correct quotient rule or correct product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = \frac{-2e^{-2x}(1-x^2) + 2xe^{-2x}}{(1-x^2)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	M1	
	Obtain a correct 3-term equation e.g. $2x^2 + 2x - 2 = 0$ or $x^2 + x = 1$	A1	From correct work only
	Solve and obtain $x = 0.618$ only	A1	From correct work only
		5	

18. 9709/32/O/N/19 Q5

Question	Answer	Marks	Guidance
5	State $4xy + 2x^2 \frac{dy}{dx}$, or equivalent, as derivative of $2x^2y$	B1	
	State $y^2 + 2xy \frac{dy}{dx}$, or equivalent, as derivative of xy^2	B1	
	Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero (or set numerator equal to zero)	*M1	$\frac{dy}{dx} = \frac{y^2 - 4xy}{2x^2 - 2xy}$
	Reject $y = 0$	B1	Allow from $y^2 - kxy = 0$
	Obtain $y = 4x$	A1	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	DM1	$8x^3 - 16x^3 = a^3$ or $\frac{y^3}{8} - \frac{y^3}{4} = a^3$
	Obtain $y = -2a$	A1	With no errors seen
		7	

19. 9709/33/O/N/19 Q4

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Question	Answer	Marks	Guidance
4(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	M1	
	Using the tan $2A$ formula, express the entire equation in terms of tan x	M1	
	Obtain a correct equation in tan x in any form	A1	
	Obtain the given form correctly	A1	AG
		4	
4(ii)	Use correct method to solve the given equation for x	M1	
	Obtain answer, e.g. $x = 26.8°$	A1	
	Obtain second answer, e.g. $x = 73.7°$ and no other	A1	Ignore answers outside the given interval
		3	

20. 9709/32/F/M/20 Q7

Question	Answer	Marks	Guidance
7(a)	State or imply $3y^2 + 6xy \frac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	
	Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Need to see $\frac{dy}{dx}$ factorised out prior to AG
	Obtain the given answer correctly	A1	AG
		4	
7(b)	Equate denominator to zero	*M1	
	Obtain $y = 2x$, or equivalent	A1	
	Obtain an equation in x or y	DM1	
	Obtain the point (1, 2)	A1	
	State the point $\left(\sqrt[3]{5}, 0\right)$	B1	Alternatively (1.71, 0).
		5	

21. 9709/31/M/J/20 Q4

Question	Answer	Marks
4(a)	Use product rule	M1
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	A1
	Equate derivative to zero and obtain an equation in one trigonometric ratio	M1
	Obtain $x = 1.43$ only	A1
		4
4(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	M1
	Show that it is a maximum point	A1
		2

22. 9709/32/M/J/20 Q4

Question	Answer	Marks
4	Use correct product rule	M1
	Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$	A1
	Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	M1
	Equate derivative to zero and obtain an equation in one trig function	M1
	Obtain $3 \sin 2x = 1$, or $3 \cos 2x = 2$ or $2 \tan 2x = 1$	A1
	Solve and obtain $x = 0.615$	A1
		6

23. 9709/33/M/J/20 Q4

Question	Answer	Marks
4(a)	Use the product rule	M1
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	M1
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right) + \frac{2x}{x^2+4}$, or equivalent	A1
		3
4(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	B1
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\dfrac{\frac{1}{2}\pi - p}{2}$	M1
	Obtain answer $p = -1$	A1
		3

24. 9709/31/O/N/20 Q3

Question	Answer	Marks	Guidance
3	State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	A1	OE
	Use correct double angle formulae	M1	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	M1	
	$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	B1	
	$\frac{dy}{dx} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	M1 A1	
	Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG

25. 9709/32/O/N/20 Q5

Question	Answer	Marks	Guidance
5(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	B1	CWO, AEF.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	M1	$y = \frac{1}{1+x^2}$
	$\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$	A1	OE
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
		3	

Question	Answer	Marks	Guidance
5(b)	Use correct product rule to obtain $\frac{d}{d\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	M1	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	A1	e.g. $\pm\left(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	A1	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	A1	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{d^2y}{dx^2}$	M1	
	Obtain correct derivative in any form	A1	$\frac{-2\left(1+x^2\right)^2 + 2\times 2x \times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	A1	e.g. $6x^2 = 2$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	A1	
		4	

26. 9709/33/O/N/20 Q3

Question	Answer	Marks	Guidance
3	State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	A1	OE
	Use correct double angle formulae	M1	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	M1	
	$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	B1	
	$\frac{dy}{dx} = \frac{\left(2 + 2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	M1 A1	
	Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG

27. 9709/31/M/J/21 Q6

Question	Answer	Marks	Guidance
6(a)	Use correct chain rule **or** correct quotient rule to differentiate x or y	M1	
	Obtain $\frac{dx}{dt} = \frac{3}{2+3t}$ **or** $\frac{dy}{dt} = \frac{2}{(2+3t)^2}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain answer $\frac{2}{3(2+3t)}$	A1	OE. Express as a simple fraction but not necessarily fully cancelled.
	Explain why this is always positive	A1	For correct gradient. e.g. x is only defined for $2+3t>0$ hence gradient >0
	Alternative method for Question 6(a)		
	Form equation in x and y only	M1	
	Obtain $y = \frac{e^x - 2}{3e^x}\left(= \frac{1}{3} - \frac{2}{3}e^{-x}\right)$	A1	OE
	Differentiate	M1	
	Obtain $y' = \frac{2}{3}e^{-x}$	A1	OE
	Explain why this is always positive	A1	
		5	

Question	Answer	Marks	Guidance
6(b)	Obtain $y = -\frac{1}{3}$ when $x = 0$	B1	
	Use a correct method to form the given tangent	M1	$\left(\frac{y+\frac{1}{3}}{x} = \frac{2}{3}\right)$
	Obtain answer $3y = 2x - 1$	A1	OE
		3	

28. 9709/32/M/J/21 Q3

Question	Answer	Marks	Guidance
3(a)	State or imply $\ln x = \ln A - y\ln 3$	B1	$\left(y = -\frac{1}{\ln 3}\ln x + \frac{\ln A}{\ln 3}\right)$
	State that the graph of y against $\ln x$ has an equation that is *linear* in y and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line	B1	Must be a correct statement. Accept if the 2 equations are written side by side with no comment. An equation with $y\ln 3$ should be compared with the form $py + q\ln x = c$.
	State that the gradient is $-\frac{1}{\ln 3}$	B1	OE. Exact answer required. ISW after a correct statement.
		3	

Question	Answer	Marks	Guidance
3(b)	Substitute $\ln x = 0$, $y = 1.3$ and use correct method to solve for A	M1	$(\ln A = 1.3\ln 3)$ Follow *their* equation in y and $\ln x$. Must be substituting $\ln x = 0$, not $x = 0$. ln 0 'used' in the solution scores M0A0.
	Obtain answer $A = 4.17$ only	A1	Must be 2 d.p. as specified in question
		2	

29. 9709/33/M/J/21 Q3

Question	Answer	Marks
3(a)	State $\dfrac{dx}{dt} = 1 + \dfrac{1}{t+2}$	B1
	Use product rule	M1
	Obtain $\dfrac{dy}{dt} = e^{-2t} - 2(t-1)e^{-2t}$	A1
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	M1
	Obtain correct answer in any simplified form, e.g. $\dfrac{(3-2t)(t+2)}{t+3}\,e^{-2t}$	A1
		5
3(b)	Equate derivative to zero and solve for t	M1
	Obtain $t = \dfrac{3}{2}$ and obtain answer $y = \dfrac{1}{2}e^{-3}$, or exact equivalent	A1
		2

30. 9709/31/O/N/21 Q3

Question	Answer	Marks	Guidance
3(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx} = e^{1-2x} - 2xe^{1-2x}$
	Equate derivative to zero and solve for x	M1	
	Obtain $x = \frac{1}{2}$ and $y = \frac{1}{2}$	A1	
		4	
3(b)	Use a correct method for determining the nature of a stationary point	M1	e.g. $\frac{d^2y}{dx^2} = -2e^{1-2x} - 2(1-2x)e^{1-2x}$
	Show that it is a maximum point	A1	
		2	

31. 9709/32/O/N/21 Q9

Question	Answer	Marks	Guidance
9(a)	State correct derivative of ye^{2x} with respect to x	B1	$2ye^{2x} + e^{2x}\frac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	B1	$2ye^x\frac{dy}{dx} + y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$	A1	Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of e^x without comment.
	Alternative method for Question 9(a)		
	Rearrange as $y = \frac{2}{e^{2x} - ye^x}$ $\Rightarrow \frac{d}{dx}\left(e^{2x} - ye^x\right) = 2e^{2x} - ye^x - e^x\frac{dy}{dx}$	B1	Other rearrangements are possible e.g. $y = 2e^{-2x} + y^2e^{-x}$ $\frac{d}{dx}\left(y^2e^{-x}\right) = 2ye^{-x}\frac{dy}{dx} - y^2e^{-x}$
	$\frac{dy}{dx} = -\frac{2}{\left(e^{2x} - ye^x\right)^2} \times \left(2e^{2x} - ye^x - e^x\frac{dy}{dx}\right)$	B1	$\Rightarrow \frac{dy}{dx} = -4e^{-x} + 2ye^{-x}\frac{dy}{dx} - y^2e^{-x}$
	Solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$	A1	Obtain the **given answer** correctly.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $ae^{3x} = b$ or $cy^3 = d$	DM1	$\left(e^{3x} = 8,\ \ y^3 = 1\right)$ SOI
	Obtain $x = \ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y = 1$	A1	
		4	

32. 9709/32/O/N/21 Q11

Question	Answer	Marks	Guidance
11(a)	Use chain rule	M1	Allow if not starting with the correct index.
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of tan x	A1	e.g. $\frac{dy}{dx} = \frac{1+\tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	B1	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
		4	

Question	Answer	Marks	Guidance
11(b)	Equate answer to part (**a**) to 1 and obtain a quartic equation in t or tan x	*M1	At least as far as $(1+\tan^2 x)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$	A1	Or equivalent horizontal form.
	Commence division by $t - 1$	DM1	As far as $t^3 + t^2 + \ldots$by long division or inspection. Allow verification by multiplying given answer by $t - 1$.
	Obtain the given answer	A1	
		4	

33. 9709/33/O/N/21 Q7

Question	Answer	Marks
7(a)	Use chain rule to differentiate LHS	*M1
	Obtain $\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$	A1
	Equate derivative of LHS to $1 - 2\frac{dy}{dx}$ and solve for $\frac{dy}{dx}$	DM1
	Obtain the given answer correctly	A1
		4
7(b)	State $x + y = 1$	B1
	Obtain or imply $x - 2y = 0$	B1
	Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$	B1
		3

34. 9709/32/F/M/22 Q4

Question	Answer	Marks	Guidance
4	State $\frac{dx}{d\theta} = \sin\theta$ or $\frac{dy}{d\theta} = -\sin\theta + \frac{1}{2}\sin 2\theta$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain correct answer in any form	A1	e.g. $\frac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta}$
	Use double angle correctly to obtain $\frac{dy}{dx}$ in terms of θ	M1	$\sin 2\theta = 2\sin\theta\cos\theta$
	Obtain the given answer with no errors seen $-2\sin^2\left(\frac{1}{2}\theta\right)$	A1	AG. Requires correct cancellation of ALL $\sin\theta$ terms and $\cos\theta = 1 - 2\sin^2\left(\frac{1}{2}\theta\right)$ seen **SC** For incorrect signs, consistent throughout max. B0, M1, A0, M1, A1
		5	

35. 9709/31/M/J/22 Q8

Question	Answer	Marks
8(a)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1
	State or imply $2y + 2x \frac{dy}{dx}$ as derivative of $2xy$	B1
	Complete differentiation and equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1
	Obtain answer $-\frac{3x^2 + 2y}{3y^2 + 2x}$	A1
		4
8(b)	Find gradient at either (0, – 2) or (– 2, 0)	M1
	Obtain answers $\frac{1}{3}$ and 3	A1 A1
	Use $\tan(A \pm B)$ formula to find $\tan\alpha$	M1
	Obtain answer $\tan\alpha = \frac{4}{3}$	A1
		5

36. 9709/32/M/J/22 Q4

Question	Answer	Marks	Guidance
4	Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	M1	e.g. two terms with one part of $\frac{dy}{dx} = p\cos^2 x \sin x \sqrt{\sin x} + q\frac{\cos^3 x \cos x}{\sqrt{\sin x}}$.
	Obtain correct derivative in any form e.g. $\frac{dy}{dx} = -3\cos^2 x \sin x \sqrt{\sin x} + \frac{\cos^3 x \cos x}{2\sqrt{\sin x}}$	A1 A1	A1 for each correct term substituted in the complete derivative.
	Equate their derivative to zero and obtain a horizontal equation with positive integer powers of $\sin x$ and/or $\cos x$ from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	M1	e.g. $-3\cos^2 x \sin^2 x + \frac{1}{2}\cos^4 x = 0$
	Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	M1	Can be awarded before the previous M1. May involve more than one trigonometric term.
	Obtain $7\cos^2 x = 6$, $7\sin^2 x = 1$, or $6\tan^2 x = 1$, or equivalent, and obtain answer $x = 0.388$	A1	CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. $22.2°$ is A0.
		6	

37. 9709/32/M/J/22 Q7

Question	Answer	Marks	Guidance
7(a)	State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2 y$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	Allow B1 B1 for $\left(3x^2 dx + \right) 6xy dx + 3x^2 dy - 3y^2 dy [= 0]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\frac{dy}{dx}$ as subject	M1	Allow if zero implied by subsequent working. Allow if recover from an extra $\frac{dy}{dx} = \ldots$ at the beginning of the left-hand side.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ correctly	A1	AG Accept y' for $\frac{dy}{dx}$.
		4	
7(b)	Equate numerator to zero	*M1	Must be using the given derivative.
	Obtain $x = -2y$, or equivalent	A1	An equation with x or y as the subject SOI.
	Use $x^3 + 3x^2 y - y^3 = 3$ to obtain an equation in x or y	DM1	$-8y^3 + 12y^3 - y^3 = 3$ or $x^3 - \frac{3}{2}x^3 + \frac{1}{8}x^3 = 3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point $(-2, 1)$ and no others from solving their cubic equation	A1	Allow if each component stated separately. ISW.
	State the point $(0, -\sqrt[3]{3})$, or equivalent from correct work	B1	Accept $(0, \sqrt[3]{-3})$, or $(0, -1.44)$ (-1.44225). Allow if each component stated separately. ISW.
		5	

38. 9709/33/M/J/22 Q4

Question	Answer	Marks	Guidance
4(a)	Use correct product rule or quotient rule, and attempt at chain rule	M1	$ke^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\dfrac{e^{4x}\sec^2 x - \tan x(ke^{4x})}{(e^{4x})^2}$ Need to see d(tan x)/dx = $\sec^2 x$ (formula sheet) and attempt at ke^{-4x}, where $k \neq 1$.
	Obtain correct derivative in any form	A1	$-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\dfrac{e^{4x}\sec^2 x - \tan x(4e^{4x})}{(e^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $ke^{-4x}\sin x\cos x\sec^2 x + ae^{-4x}\sec^2 x$ or $ke^{-4x}\dfrac{\sin x\cos x}{\cos x\cos x} + ae^{-4x}\sec^2 x$ or $\sec^2 x(ke^{-4x}\sin x\cos x + ae^{-4x})$ Allow $\dfrac{1}{\cos^2 x}$ instead of $\sec^2 x$	M1	Need to use $\dfrac{\tan x}{\sec^2 x} = \sin x\cos x$ or $\tan x = \dfrac{\sin x}{\cos x}.\dfrac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	A1	At least one line of trigonometric working is required from $-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ to given answer $\sec^2 x(1 - 2\sin 2x)e^{-4x}$ with elements in any order. If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0.
		4	

Question	Answer	Marks	Guidance
4(b)	Equate derivative to zero and use correct method to solve for x	M1	$\sin 2x = \dfrac{1}{2}$, hence $x = \dfrac{1}{2}\sin^{-1}\dfrac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x = \dfrac{1}{12}\pi$	A1	[0.262 M1 A0]
	Obtain second answer, e.g. $\dfrac{5}{12}\pi$ and no other in the given interval	A1 FT	FT $\dfrac{\pi - \textit{their}\ 2x}{2}$ if exact values; x must be $< \dfrac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°. **SC** No values found for a and b in **4(a)** but chooses values in **4(b)**: max **M1** for x.
		3	

39. 9709/33/M/J/22 Q6

Question	Answer	Marks	Guidance
6(a)	Use chain rule at least once	**M1**	Needs $\frac{dy}{dt}=\frac{1}{\tan t}\frac{d}{dt}(\tan t)$ or $\frac{dx}{dt}=(-1)(\cos^{-2}t)\frac{d}{dt}(\cos t)$. BOD if + and (– 1) (– 1) not seen. $\frac{dx}{dt}=\sec t\tan t$ (from List of Formulae MF19) M1 A1. If $\frac{dx}{dt}=-\sec t\tan t$ M1 A0.
	Obtain $\frac{dx}{dt}=\sec t\tan t$	**A1**	OE e.g. $\sin t(\cos t)^{-2}$. If e.g. $\frac{dx}{dt}=\sec x\tan x$ or $\sec\theta\tan\theta$ or $\sec t\tan x$, condone recovery on next line.
	Obtain $\frac{dy}{dt}=\frac{\sec^2 t}{\tan t}$	**A1**	OE e.g. $\frac{1}{\sin t\cos t}$. If e.g. $\frac{dy}{dt}=\frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line. Only penalise notation errors **once** in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery.
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	**M1**	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	**A1**	AG After $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ used, any notation error A0. Must cancel cos *t* correctly.
		5	

Question	Answer	Marks	Guidance
6(b)	State or imply $t=\frac{1}{4}\pi$ when $y=0$	**B1**	
	Form the equation of the tangent at $y=0$ or find c	**M1**	$x=\sqrt{2}$, $\frac{dy}{dx}=\sqrt{2}$ and $y=0$, *their* coordinates and gradient used in $y=mx+c$.
	Obtain answer $y=\sqrt{2}x-2$	**A1**	OE e.g. $y=\sqrt{2}(x-\sqrt{2})$ ISW. Allow $y=1.41x-2[.00]$ or $1.41(x-1.41)$.
		3	

40. 9709/31/O/N/22 Q7

Question	Answer	Marks	Guidance
7(a)	Use correct product or quotient rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x\sin x\cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x\sec^2 x\tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1}\left(\sqrt[3]{\frac{\cos a + 2a\sin a}{12}}\right)$	A1	AG
		3	
7(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	M1	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	A1	e.g. $\begin{matrix}\cos 0.9 = 0.622 > 0.553 \\ \cos 1 = 0.540 < 0.570\end{matrix}$ or $\begin{matrix}0.9 < 0.985 \\ 1 > 0.964\end{matrix}$ or $\begin{matrix}0.0846 > 0 \\ -0.0358 < 0\end{matrix}$ or could be looking at values of the gradient 8.46 & 14.1
		2	

41. 9709/33/O/N/22 Q4

Question	Answer	Marks
4	State or imply $\frac{dx}{dt} = 2 - \sec^2 t$ or $\frac{dy}{dt} = 2\frac{\cos 2t}{\sin 2t}$	B1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1
	Obtain correct answer in any form	A1
	Use double angle formula to express derivative in terms of cos x and sin x	M1
	Obtain the given answer correctly	A1
		5

42. 9709/33/O/N/22 Q8

Question	Answer	Marks
8(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative at $x = p$ to zero and obtain the given equation	A1
		3
8(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p = 2.5$ and $p = 3$	M1
	Complete the argument with correct calculated values	A1
		2

43. 9709/32/F/M/23 Q5

PUBLISHED

Question	Answer	Marks	Guidance
5(a)	Obtain $\frac{dx}{dt} = e^{2t} + 2te^{2t}$	B1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	M1	$\frac{dy}{dx} = \frac{2t+1}{e^{2t}(1+2t)}$.
	Obtain the given answer $\frac{dy}{dx} = e^{-2t}$	A1	AG Need to see $e^{2t}(1 + 2t)$ in denominator.
		3	
5(b)	Obtain $x = -e^{-2}$ or $-\frac{1}{e^2}$ and $y = 3$ at $t = -1$	B1	
	Obtain gradient of normal $= -e^{-2}$ or $-\frac{1}{e^2}$	B1	
	$x = 0$ substituted into equation of normal or use of gradients to give $y = 3 - \frac{1}{e^4}$ with no errors	B1	Equation of normal $y - 3 = -e^{-2}\left(x - -e^{-2}\right)$. AG **SC** Decimals **B0 B1 B0** $-$ 0.135 .
		3	

44. 9709/31/M/J/23 Q5

PUBLISHED

Question	Answer	Marks	Guidance
5(a)	State or imply $2xy + x^2\frac{dy}{dx}$ as derivative of x^2y	B1	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{dy}{dx}$ as derivative of ay^2	B1	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working	A1	AG
		4	
5(b)	State or imply $2ay - x^2 = 0$	*M1	
	Substitute into equation of curve to obtain equation in x and a or in y and a	DM1	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	A1	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	A1	e.g. $(-2a, 2a)$.
		4	**SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

45. 9709/32/M/J/23 Q7

Question	Answer	Marks	Guidance
7(a)	State or imply $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	Allow y' for $\frac{dy}{dx}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x\frac{dy}{dx} + 4y$ as the derivative of $4xy$	B1	Accept $\frac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Allow an extra $\frac{dy}{dx}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{dy}{dx} = -\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial y}$
	Obtain $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$	A1	AG – must come from correct working. The position of the negative must be clear.
		4	

Question	Answer	Marks	Guidance
7(b)	Equate $\frac{dy}{dx}$ to –2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

46. 9709/33/M/J/23 Q4

Question	Answer	Marks	Guidance
4	State $\frac{dy}{d\theta} = 1 - 2\sin\theta$	B1	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\frac{dy}{dx}$ for final A1.
	Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2 - \sin\theta)^{-1}$	M1	Incorrect formula seen M0 A0 otherwise BOD.
	Obtain $\frac{dx}{d\theta} = \frac{-(2-\sin\theta)\sin\theta + \cos^2\theta}{(2-\sin\theta)^2}$ o.e.	A1	$-\sin\theta(2 - \sin\theta)^{-1} - \cos\theta(2 - \sin\theta)^{-2}(-\cos\theta)$ or equivalent.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	$\left(\frac{dy}{dx} = (1 - 2\sin\theta) \div \frac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
	Obtain $\frac{dy}{dx} = (2 - \sin\theta)^2$.	A1	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta + \sin^2\theta = 1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
		5	

47. 9709/31/O/N/23 Q6

Question	Answer	Marks	Guidance
6(a)	State correct derivative of x **or** y with respect to t	B1	$\frac{dx}{dt}=\frac{1}{2}t^{-\frac{1}{2}}$, $\frac{dy}{dt}=\frac{1}{t}$.
	Use $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$	M1	Use correct chain rule.
	Obtain answer $\frac{dy}{dx}=\frac{2}{\sqrt{t}}$	A1	Or simplified equivalent e.g $2t^{-\frac{1}{2}}$ or $\frac{2\sqrt{t}}{t}$.
		3	
6(b)	State or imply *their* $\frac{dy}{dx}=\frac{1}{2}$	M1	
	Obtain $\sqrt{t}=4$	A1	Or equivalent.
	Obtain answer (7, ln 16)	A1	Or exact equivalent. Can state the two components separately.
		3	

48. 9709/32/O/N/23 Q2

Question	Answer	Marks	Guidance
2	Obtain $\frac{dx}{dt}=\frac{2}{t}\ln t$	B1	Any equivalent form.
	Obtain $\frac{dy}{dt}=-2te^{2-t^2}$	B1	Any equivalent form.
	$\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ and substitute $t=e$	M1	Correct use of chain rule for $\frac{dy}{dx}\left(\frac{-2e^2e^{2-e^2}}{2\ln e}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
	Obtain $-e^{4-e^2}$	A1	ISW Accept $-0.0337(405..)$. Accept $-e^4e^{-e^2}$, $\frac{-e^4}{e^{e^2}}$ and $-e^2e^{2-e^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
		4	

49. 9709/33/O/N/23 Q5

Question	Answer	Marks	Guidance
5	Use correct product or quotient rule	M1	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if $\pm 2x$ seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
	Obtain correct derivative in any form, e.g. $\frac{6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}}{(1-x^2)^2}$	A1	If $6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}=0$ from the start, with no wrong formula seen, award M1A1.
	Equate derivative (or its numerator) to zero and solve for x	M1	$6x-6x^3+2x=0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x=0$ the x must factorise out
	Obtain the point $(0, e^{-1})$ or exact equivalent	A1	Or for all three x coordinates found 0, $\pm\frac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x=0, y=e^{-1}$ ISW for last 3 marks.
	Obtain the point $\left(\frac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	Allow $\sqrt{(4/3)}$.
	Obtain the point $\left(-\frac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	
		6	

50. 9709/32/F/M/24 Q6

Question	Answer	Marks	Guidance
6(a)	State or imply $4y\frac{dy}{dx}$ as the derivative of $2y^2$	B1	**SC** If $\frac{dy}{dx}$ introduced instead of $\frac{d}{dx}$ then allow **B1** for both, followed by correct method **M1** Max 2.
	State or imply $3y+3x\frac{dy}{dx}$ as the derivative of $3xy$	B1	Allow extra $\frac{dy}{dx}$ = correct expression to collect all marks if correct.
	Complete the differentiation, all 4 terms, isolate 2 $\frac{dy}{dx}$ terms on LHS or bracket $\frac{dy}{dx}$ terms and solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2x-3y-1}{4y+3x}$	A1	Answer Given – need to have seen $4y\frac{dy}{dx}+3x\frac{dy}{dx}$ $=2x-3y-1$ or $(4y+3x)\frac{dy}{dx}-2x+3y=-1$. Need to see $=2x$ or $=0$ consistently throughout otherwise **M1 A0**. No recovery allowed. When all terms are included then must be an equation.
		4	Allow all marks if using dx and dy.
6(b)	Equate numerator to zero, obtaining $2x=3y+1$ or $3y=2x-1$ and form equation in x only or y only from $2y^2+3xy+x=x^2$	M1*	e.g. $\frac{2}{9}(2x-1)^2+x(2x-1)+x=x^2$ or $2y^2+\frac{3}{2}(1+3y)y+\frac{1}{2}(1+3y)=\frac{1}{4}(1+3y)^2$. Allow errors.
	Obtain $\frac{2}{9}(2x-1)^2=-x^2$ or a 3 term quadratic in one unknown and try to solve. If errors in quadratic formulation allow solution, applying usual rules for solution of quadratic equation, and allow **M1**	DM1	e.g. $17x^2-8x+2=0$ $(b^2-4ac=-72)$ or $17y^2+6y+1=0$ $(b^2-4ac=-32)$. $x=4/17\pm(3\sqrt{2}/17)i$, $y=-3//17\pm(2\sqrt{2}/17)i$.
	Conclude that the equation has no [real] roots	A1	Given Answer. CWO
		3	