1. 9709/32/F/M/18 Q3

(i) Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

2. 9709/31/M/J/18 Q2

(i) Given that $\sin(x - 60°) = 3\cos(x - 45°)$, find the exact value of $\tan x$. [4]

(ii) Hence solve the equation $\sin(x - 60°) = 3\cos(x - 45°)$, for $0° < x < 360°$. [2]

3. 9709/32/M/J/18 Q2

Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45°) = 2$, for $0° < \theta < 180°$. [5]

4. 9709/32/M/J/18 Q4

(i) Show that $\frac{2\sin x - \sin 2x}{1 - \cos 2x} \equiv \frac{\sin x}{1 + \cos x}$. [4]

5. 9709/33/M/J/18 Q5

(i) By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \tfrac{3}{4}\sin^2 2x. \qquad [4]$$

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \tfrac{2}{3},$$

for $0° < x < 180°$. [4]

6. 9709/33/M/J/18 Q7

(i) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan\alpha$. [3]

7. 9709/31/O/N/18 Q6

(i) Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

8. 9709/32/O/N/18 Q2

Showing all necessary working, solve the equation $\sin(\theta - 30°) + \cos\theta = 2\sin\theta$, for $0° < \theta < 180°$.
[4]

9. 9709/33/O/N/18 Q6

(i) Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

10. 9709/32/F/M/19 Q3

(i) Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

11. 9709/31/M/J/19 Q4

By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

12. 9709/31/M/J/19 Q6

(i) By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$. [4]

13. 9709/32/M/J/19 Q3

Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

14. 9709/33/M/J/19 Q3

Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan\theta$. [3]

15. 9709/31/O/N/19 Q9

(i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

16. 9709/32/O/N/19 Q4

(i) Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

17. 9709/33/O/N/19 Q4

(i) By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3\cot x$ can be written in the form $\tan^4 x - 12\tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

18. 9709/32/F/M/20 Q5

(a) Show that $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 2\cot 2x$. [4]

(b) Hence solve the equation $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 4$, for $0 < x < \pi$. [3]

19. 9709/31/M/J/20 Q3

Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

20. 9709/31/M/J/20 Q7

Let $f(x) = \dfrac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

21. 9709/32/M/J/20 Q4

A curve has equation $y = \cos x \sin 2x$.

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

22. 9709/32/M/J/20 Q5

(a) Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

23. 9709/33/M/J/20 Q5

By first expressing the equation

$$\tan\theta\tan(\theta+45^\circ)=2\cot 2\theta$$

as a quadratic equation in $\tan\theta$, solve the equation for $0^\circ<\theta<90^\circ$. [6]

24. 9709/31/O/N/20 Q6

(a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

25. 9709/32/O/N/20 Q4

(a) Show that the equation $\tan(\theta + 60°) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0.$$ [3]

(b) Hence solve the equation $\tan(\theta + 60°) = 2\cot\theta$, for $0° < \theta < 180°$. [3]

26. 9709/33/O/N/20 Q6

(a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

27. 9709/32/F/M/21 Q3

By first expressing the equation $\tan(x + 45°) = 2\cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0° < x < 180°$. [6]

28. 9709/32/F/M/21 Q5

(a) Express $\sqrt{7}\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$, for $0° < \theta < 180°$. [5]

29. 9709/31/M/J/21 Q3

(a) Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

(b) Hence solve the equation

$$\cos(x - 30°) = 2\sin(x + 30°),$$

for $0° < x < 360°$. [2]

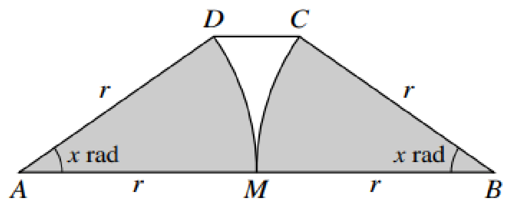
30. 9709/31/M/J/21 Q4

(a) Prove that $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2\theta$. [2]

31. 9709/32/M/J/21 Q6

(a) Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan\theta$. [3]

32. 9709/32/M/J/21 Q10



The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M, the midpoint of AB.

(a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x)\sin x$. [3]

(b) Verify by calculation that x lies between 0.5 and 0.7. [2]

33. 9709/33/M/J/21 Q5

(a) By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

34. 9709/31/O/N/21 Q2

(a) Express $5\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$. [2]

35. 9709/31/O/N/21 Q5

(a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$. [3]

b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places. [3]

36. 9709/32/O/N/21 Q6

(a) Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\tfrac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$ [3]

37. 9709/32/O/N/21 Q8

(a) By first expanding $(\cos^2\theta + \sin^2\theta)^2$, show that

$$\cos^4\theta + \sin^4\theta \equiv 1 - \tfrac{1}{2}\sin^2 2\theta.$$ [3]

(b) Hence solve the equation

$$\cos^4\theta + \sin^4\theta = \tfrac{5}{9},$$

for $0° < \theta < 180°$. [4]

38. 9709/33/O/N/21 Q5

Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

39. 9709/33/O/N/21 Q6

(a) By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

40. 9709/32/F/M/22 Q5

The angles α and β lie between 0° and 180° and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan\alpha = 3\tan\beta.$$

Find the possible values of α and β. [6]

41. 9709/31/M/J/22 Q3

Solve the equation $2\cot 2x + 3\cot x = 5$, for $0° < x < 180°$. [6]

42. 9709/32/M/J/22 Q3

Solve the equation $3\cos 2\theta = 3\cos\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

43. 9709/33/M/J/22 Q2

Solve the equation $\cos(\theta - 60^\circ) = 3\sin\theta$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

44. 9709/31/O/N/22 Q4

Solve the equation $\tan(x + 45°) = 2\cot x$ for $0° < x < 180°$. [5]

45. 9709/31/O/N/22 Q6

(a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$. [4]

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [3]

46. 9709/32/O/N/22 Q3

The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

47. 9709/32/O/N/22 Q4

(a) Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

48. 9709/33/O/N/22 Q7

(a) Show that the equation $\sqrt{5}\sec x + \tan x = 4$ can be expressed as $R\cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

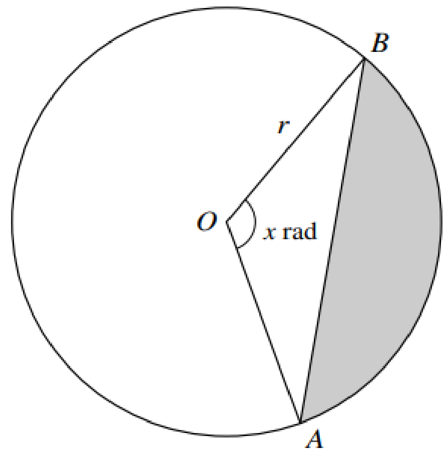
(b) Hence solve the equation $\sqrt{5}\sec 2x + \tan 2x = 4$, for $0° < x < 180°$. [4]

49. 9709/32/F/M/23 Q6

(a) Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. [4]

50. 9709/32/F/M/23 Q7



The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$. [4]

(b) Show by calculation that the root of the equation in **(a)** lies between 2 and 2.5. [2]

51. 9709/31/M/J/23 Q4

(a) Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

52. 9709/32/M/J/23 Q4

Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

53. 9709/32/M/J/23 Q6

The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

54. 9709/33/M/J/23 Q6

(a) Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

55. 9709/31/O/N/23 Q5

(a) Given that

$$\sin\left(x+\tfrac{1}{6}\pi\right)-\sin\left(x-\tfrac{1}{6}\pi\right)=\cos\left(x+\tfrac{1}{3}\pi\right)-\cos\left(x-\tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

(b) Hence find the exact roots of the equation

$$\sin\left(x+\tfrac{1}{6}\pi\right)-\sin\left(x-\tfrac{1}{6}\pi\right)=\cos\left(x+\tfrac{1}{3}\pi\right)-\cos\left(x-\tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

56. 9709/32/O/N/23 Q7

(a) By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence solve the equation

$$\cos 3\theta + \cos\theta\cos 2\theta = \cos^2\theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

57. 9709/33/O/N/23 Q6

(a) Show that the equation $\cot^2\theta + 2\cos 2\theta = 4$ can be written in the form

$$4\sin^4\theta + 3\sin^2\theta - 1 = 0.$$ [3]

(b) Hence solve the equation $\cot^2\theta + 2\cos 2\theta = 4$, for $0° < \theta < 360°$. [3]

58. 9709/32/F/M/24 Q8

(a) Express $3\sin x + 2\sqrt{2}\cos\left(x + \frac{1}{4}\pi\right)$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. State the exact value of R and give α correct to 3 decimal places. [4]

(b) Hence solve the equation

$$6\sin\tfrac{1}{2}\theta + 4\sqrt{2}\cos\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\pi\right) = 3$$

for $-4\pi < \theta < 4\pi$. [5]