### 1. 9709/32/F/M/18 Q3

| Question | Answer | Marks |
|---|---|---|
| 3(i) | State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$ | B1 |
| | Substitute in $\frac{1}{2}(\cos 4x + \cos 2x)$ | M1 |
| | Obtain the given identity correctly **AG** | A1 |
| | | 3 |

### 2. 9709/31/M/J/18 Q2

| Question | Answer | Marks |
|---|---|---|
| 2(i) | Use trig formulae and obtain an equation in $\sin x$ and $\cos x$ | M1* |
| | Obtain a correct equation in any form | A1 |
| | Substitute exact trig ratios and obtain an expression for $\tan x$ | M1(dep*) |
| | Obtain answer $\tan x = \dfrac{-(6+\sqrt{6})}{(6-\sqrt{2})}$ or equivalent | A1 |
| | | 4 |
| 2(ii) | State answer, e.g. 118.5° | B1 |
| | State second answer, e.g. 298.5° | B1ft |
| | | 2 |

### 3. 9709/32/M/J/18 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use correct $\tan(A \pm B)$ formula and obtain an equation in $\tan\theta$ | M1 | $\dfrac{1}{\tan\theta} + \dfrac{1-\tan\theta\tan 45}{\tan\theta+\tan 45} = 2$ Allow M1 with $\tan 45°$ $= \dfrac{1}{\tan\theta} + \dfrac{1-\tan\theta}{\tan\theta+1}$ |
| | Obtain a correct equation in any form | A1 | With values substituted |
| | Reduce to $3\tan^2\theta = 1$, or equivalent | A1 | |
| | Obtain answer $x = 30°$ | A1 | One correct solution |
| | Obtain answer $x = 150°$ | A1 | Second correct solution and no others in range |
| | *OR*: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ | M1A1 | |
| | Reduce to $\tan^2\theta = \frac{1}{3}, \sin^2\theta = \frac{1}{4}, \cos^2\theta = \frac{3}{4}$ or $\cot^2\theta = 3$ etc. | A1 | |
| | | 5 | |

### 4. 9709/32/M/J/18 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$ | M1 | $\dfrac{2\sin x - 2\sin x \cos x}{1 - \left(2\cos^2 x - 1\right)}$ |
| | Obtain a correct expression | A1 | |
| | Complete method to get correct denominator e.g. by factorising to remove a factor of $1 - \cos x$ | M1 | |
| | Obtain the given RHS correctly<br>*OR (working R to L):* | A1 | |
| | $\dfrac{\sin x}{1+\cos x} \times \dfrac{1-\cos x}{1-\cos x} = \dfrac{\sin x - \sin x \cos x}{1-\cos^2 x}$<br>$= \dfrac{2\sin x - 2\sin x \cos x}{2 - 2\cos^2 x}$ | M1A1 | Given answer so check working carefully |
| | $= \dfrac{2\sin x - \sin 2x}{1 - \cos 2x}$ | M1A1 | |
| | | 4 | |

### 5. 9709/33/M/J/18 Q5

| Question | Answer | Marks |
|---|---|---|
| 5(i) | Attempt cubic expansion and equate to 1 | M1 |
| | Obtain a correct equation | A1 |
| | Use Pythagoras and double angle formula in the expansion | M1 |
| | Obtain the given result correctly | A1 |
| | Total: | 4 |
| 5(ii) | Use the identity and carry out a method for finding a root | M1 |
| | Obtain answer 20.9° | A1 |
| | Obtain a second answer, e.g. 69.1° | A1FT |
| | Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval | A1FT |
| | Total: | 4 |

### 6. 9709/33/M/J/18 Q7

| Question | Answer | Marks |
|---|---|---|
| 7(i) | State answer $R = \sqrt{5}$ | B1 |
| | Use trig formulae to find $\tan \alpha$ | M1 |
| | Obtain $\tan \alpha = 2$ | A1 |
| | Total: | 3 |

### 7. 9709/31/O/N/18 Q6

| Question | Answer | Marks |
|---|---|---|
| 6(i) | Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$ | B1 |
| | State $R = 2$ | B1 |
| | Use trig formulae to obtain $\alpha$ | M1 |
| | Obtain $\alpha = 30°$ with no errors seen | A1 |
| | | 4 |

| Question | Answer | Marks |
|---|---|---|
| 6(ii) | Evaluate $\sin^{-1}\left(\dfrac{\sqrt{2}}{R}\right)$ | B1ft |
| | Carry out a correct method to find a value of $x$ in the given interval | M1 |
| | Obtain answer $x = 75°$ | A1 |
| | Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.] | A1ft |
| | | 4 |

### 8. 9709/32/O/N/18 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$ | M1* | Condone sign error in expansion and/or omission of "$+\cos\theta$" $\sin\theta\cos 30° - \cos\theta\sin 30° + \cos\theta = 2\sin\theta$ |
| | Obtain an equation in $\tan\theta$ | M1(dep*) | e.g. $\tan\theta = \dfrac{1-\sin 30°}{2-\cos 30°}$ Can be implied by correct answer following correct expansion. Otherwise need to see working |
| | Obtain $\tan\theta = 1/(4-\sqrt{3})$, or equivalent | A1 | $\dfrac{4+\sqrt{3}}{13}$, 0.4409.... (2 s.f or better) |
| | Obtain final answer $\theta = 23.8°$ and no others in range | A1 | At least 3 sf (23.7939....) ignore extra values outside range |
| | | 4 | |

## 9. 9709/33/O/N/18 Q6

| Question | Answer | Marks |
|---|---|---|
| 6(i) | Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$ | B1 |
| | State $R = 2$ | B1 |
| | Use trig formulae to obtain $\alpha$ | M1 |
| | Obtain $\alpha = 30°$ with no errors seen | A1 |
| | | 4 |

| Question | Answer | Marks |
|---|---|---|
| 6(ii) | Evaluate $\sin^{-1}\left(\dfrac{\sqrt{2}}{R}\right)$ | B1ft |
| | Carry out a correct method to find a value of $x$ in the given interval | M1 |
| | Obtain answer $x = 75°$ | A1 |
| | Obtain a second answer e.g. $x = 165°$ and no others <br> [Treat answers in radians as a misread. Ignore answers outside the given interval.] | A1ft |
| | | 4 |

## 10. 9709/32/F/M/19 Q3

| Question | Answer | Marks |
|---|---|---|
| 3(i) | Use trig formulae and obtain an equation in $\sin\theta$ and $\cos\theta$ | M1 |
| | Obtain a correct equation in any form | A1 |
| | Substitute exact trig ratios and obtain an expression for $\tan\theta$ | M1 |
| | Obtain answer $\tan\theta = \dfrac{2\sqrt{2}-1}{1-\sqrt{6}}$, or equivalent | A1 |
| | | 4 |
| 3(ii) | State answer, e.g. $\theta = 128.4°$ | B1 |
| | State second answer, e.g. $\theta = 308.4°$ | B1 ft |
| | | 2 |

## 11. 9709/31/M/J/19 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Use correct trig formula and obtain an equation in $\tan\theta$ | M1 | Allow with $45°$ e.g. $\dfrac{1}{\tan\theta}-\dfrac{1}{\frac{\tan\theta+\tan45°}{1-\tan\theta\tan45°}}=3$ |
| | Obtain a correct horizontal equation in any form | A1 | e.g. $1+\tan\theta-\tan\theta(1-\tan\theta)=3\tan\theta(1+\tan\theta)$ |
| | Reduce to $2\tan^2\theta+3\tan\theta-1=0$ | A1 | or 3-term equivalent |
| | Solve 3-term quadratic and find a value of $\theta$ | M1 | Must see working if using an incorrect quadratic |
| | Obtain answer $15.7°$ | A1 | One correct solution (degrees to at least 3 sf) |
| | Obtain answer $119.(3)°$ | A1 | Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1 |
| | | 6 | |

## 12. 9709/31/M/J/19 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | State correct expansion of $\sin(2x+x)$ | B1 | |
| | Use trig formulae and Pythagoras to express $\sin3x$ in terms of $\sin x$ | M1 | |
| | Obtain a correct expression in any form | A1 | e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$ |
| | Obtain $\sin3x\equiv3\sin x-4\sin^3 x$ correctly     AG | A1 | Accept $=$ for $\equiv$ |
| | | 4 | |

## 13. 9709/32/M/J/19 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use correct trig formulae to obtain an equation in $\tan\theta$ or equivalent (e.g all in $\sin\theta$ or all in $\cos\theta$) | *M1 | $\dfrac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\dfrac{\cot^2\theta-1}{2\cot\theta}=\dfrac{2}{\cot\theta}$ |
| | Obtain a correct simplified equation | A1 | $5\tan^2\theta=1$ or $\sin^2\theta=\dfrac{1}{6}$ or $\cos^2\theta=\dfrac{5}{6}$ |
| | Solve for $\theta$ | DM1 | Dependent on the first M1 |
| | Obtain answer $24.1°$ (or $155.9°$) | A1 | One correct in range to at least 3 sf |
| | Obtain second answer | A1 | FT $180°-their$ $24.1°$ and no others in range. Correct to at least 3 sf. Accept $156°$ but not $156.0$ Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1 |
| | | 5 | |

## 14. 9709/33/M/J/19 Q3

| Question | Answer | Marks |
|---|---|---|
| 3(i) | Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$ | M1 |
| | Obtain a correct expression | A1 |
| | Obtain the given answer | A1 |
| | | 3 |

## 15. 9709/31/O/N/19 Q9

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(i) | Use $\cos(A + B)$ formula to express $\cos3x$ in terms of trig functions of $2x$ and $x$ | M1 | |
| | Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only | M1 | |
| | Obtain a correct expression in terms of $\cos x$ in any form | A1 | |
| | Obtain $\cos3x \equiv 4\cos^3 x - 3\cos x$ | A1 | AG |
| | | 4 | |
| 9(ii) | Use identity and solve cubic $4\cos^3 x = -1$ for $x$ | M1 | $\cos x = -0.6299...$ |
| | Obtain answer 2.25 and no other in the interval | A1 | Accept $0.717\pi$<br>M1A0 for 129.0° |
| | | 2 | |

## 16. 9709/32/O/N/19 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | State $R = \sqrt{7}$ | B1 | |
| | Use correct trig formulae to find $\alpha$ | M1 | e.g. $\tan\alpha = \dfrac{1}{\sqrt{6}}$, $\sin\alpha = \dfrac{1}{\sqrt{7}}$, or $\cos\alpha = \dfrac{\sqrt{6}}{\sqrt{7}}$ |
| | Obtain $\alpha = 22.208°$ | A1 | ISW |
| | | 3 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | Evaluate $\sin^{-1}\left(\dfrac{2}{\sqrt{7}}\right)$ to at least 1 d.p. | B1FT | 49.107° to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their R* |
| | | | SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta - 2\sqrt{6}\tan\theta + 1 = 0$ |
| | Use correct method to find a value of $\theta$ in the interval | M1 | Must get to $\theta$ |
| | Obtain answer, e.g. 13.4° | A1 | Accept correct over-specified answers. 13.449..., 54.3425... |
| | Obtain second answer, e.g. 54.3° and no extras in the given interval | A1 | Ignore answers outside the given interval. |
| | | 4 | |

## 17. 9709/33/O/N/19 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan $x$ | M1 | |
| | Using the tan $2A$ formula, express the entire equation in terms of tan $x$ | M1 | |
| | Obtain a correct equation in tan $x$ in any form | A1 | |
| | Obtain the given form correctly | A1 | AG |
| | | 4 | |
| 4(ii) | Use correct method to solve the given equation for $x$ | M1 | |
| | Obtain answer, e.g. $x = 26.8°$ | A1 | |
| | Obtain second answer, e.g. $x = 73.7°$ and no other | A1 | Ignore answers outside the given interval |
| | | 3 | |

## 18. 9709/32/F/M/20 Q5

| Question | Answer | Marks |
|---|---|---|
| 5(a) | Express LHS correctly as a single fraction | B1 |
| | Use $\cos(A \pm B)$ formula to simplify the numerator | M1 |
| | Use sin $2A$ formula to simplify the denominator | M1 |
| | Obtain the given result. | A1 |
| | | 4 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | Obtain an equation in $\tan 2x$ and use correct method to solve for $x$ | M1 | |
| | Obtain answer, e.g. 0.232 | A1 | |
| | Obtain second answer, e.g. 1.80 | A1 | Ignore answers outside the given interval. |
| | | 3 | |

## 19. 9709/31/M/J/20 Q3

| Question | Answer | Marks |
|---|---|---|
| 3 | Use tan $(A \pm B)$ formula and obtain an equation in tan $\theta$ | M1 |
| | Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan $\theta$ in any correct form | A1 |
| | Reduce the equation to $3\tan^2 \theta + 4\tan \theta - 1 = 0$, or equivalent | A1 |
| | Solve a 3-term quadratic for tan $\theta$ | M1 |
| | Obtain a correct answer, e.g. 12.1° | A1 |
| | Obtain a second correct answer, e.g. 122.9°, and no others in the given interval | A1 |
| | | 6 |

## 20. 9709/31/M/J/20 Q7

| Question | Answer | Marks |
|----------|--------|-------|
| 7(a) | Use quotient or product rule | M1 |
| | Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$ | A1 |
| | Use Pythagoras to simplify the derivative | M1 |
| | Justify the given statement | A1 |
| | | 4 |

## 21. 9709/32/M/J/20 Q4

| Question | Answer | Marks |
|----------|--------|-------|
| 4 | Use correct product rule | M1 |
| | Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$ | A1 |
| | Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$ | M1 |
| | Equate derivative to zero and obtain an equation in one trig function | M1 |
| | Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$ | A1 |
| | Solve and obtain $x = 0.615$ | A1 |
| | | 6 |

## 22. 9709/32/M/J/20 Q5

| Question | Answer | Marks |
|----------|--------|-------|
| 5(a) | State $R = \sqrt{7}$ | B1 |
| | Use trig formulae to find $\alpha$ | M1 |
| | Obtain $\alpha = 57.688°$ | A1 |
| | | 3 |
| 5(b) | Evaluate $\cos^{-1}\left(\dfrac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) <br> (**FT** is on *their R*) | B1 FT |
| | Use correct method to find a value of $\theta$ in the interval | M1 |
| | Obtain answer, e.g. 5.1° | A1 |
| | Obtain second answer, e.g.117.3°, only | A1 |
| | | 4 |

### 23. 9709/33/M/J/20 Q5

| Question | Answer | Marks |
|---|---|---|
| 5 | Use tan $2A$ formula to express RHS in terms of tan $\theta$ | M1 |
| | Use tan $(A \pm B)$ formula to express LHS in terms of tan $\theta$ | M1 |
| | Using tan $45° = 1$, obtain a correct horizontal equation in any form | A1 |
| | Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$ | A1 |
| | Solve a 3-term quadratic and find a value of $\theta$ | M1 |
| | Obtain answer $\theta = 26.6°$ and no other | A1 |
| | | 6 |

### 24. 9709/31/O/N/20 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | State $R = \sqrt{15}$ | B1 | |
| | Use trig formulae to find $\alpha$ | M1 | $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \dfrac{3}{\sqrt{6}}$ quoted then allow M1 |
| | Obtain $\alpha = 50.77$ | A1 | Must be 2 d.p.<br>If radians 0.89   A0 MR |
| | | 3 | |
| 6(b) | Evaluate $\beta = \cos^{-1}\dfrac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.) | B1 FT | The FT is on incorrect $R$.<br>$\dfrac{x}{3} = \beta - \alpha$  [−2.9° and −301.7°] |
| | Use correct method to find a value of $\dfrac{x}{3}$ in the interval | M1 | Needs to use $\dfrac{x}{3}$ |
| | Obtain answer rounding to $x = 301.6°$ to $301.8°$ | A1 | |
| | Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval | A1 | |
| | | 4 | |

### 25. 9709/32/O/N/20 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$ | M1 | e.g. $\dfrac{\tan\theta + \tan 60°}{1 - \tan\theta\tan 60°} = \dfrac{2}{\tan\theta}$ |
| | Use $\tan 60° = \sqrt{3}$ and obtain a correct horizontal equation in any form | A1 | e.g. $\tan\theta\left(\tan\theta + \sqrt{3}\right) = 2\left(1 - \sqrt{3}\tan\theta\right)$ |
| | Reduce to $\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$ correctly | A1 | AG |
| | | 3 | |
| 4(b) | Solve the given quadratic to obtain a value for $\theta$ | M1 | $\left(\tan\theta = \dfrac{-3\sqrt{3} \pm \sqrt{35}}{2} = 0.3599, -5.556\right)$ |
| | Obtain one correct answer e.g. $\theta = 19.8°$ | A1 | Accept 1d.p. or better.<br>If over-specified must be correct. 19.797...., 100.2029... |
| | Obtain second correct answer $\theta = 100.2°$ and no others in the given interval | A1 | Ignore answers outside the given interval. |
| | | 3 | |

### 26. 9709/33/O/N/20 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | State $R = \sqrt{15}$ | B1 | |
| | Use trig formulae to find $\alpha$ | M1 | $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \dfrac{3}{\sqrt{6}}$ quoted then allow M1 |
| | Obtain $\alpha = 50.77$ | A1 | Must be 2 d.p. If radians 0.89   A0 MR |
| | | 3 | |
| 6(b) | Evaluate $\beta = \cos^{-1}\dfrac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.) | B1 FT | The FT is on incorrect $R$. $\dfrac{x}{3} = \beta - \alpha$  [−2.9° and −301.7°] |
| | Use correct method to find a value of $\dfrac{x}{3}$ in the interval | M1 | Needs to use $\dfrac{x}{3}$ |
| | Obtain answer rounding to $x = 301.6°$ to $301.8°$ | A1 | |
| | Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval | A1 | |
| | | 4 | |

### 27. 9709/32/F/M/21 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use correct trig formulae to obtain an equation in $\tan x$ | *M1 | |
| | Using $\tan 45° = 1$, obtain a horizontal equation in $\tan x$ in any form | DM1 | |
| | Reduce the equation to $\tan^2 x + \tan x - 1 = 0$, or 3-term equivalent | A1 | |
| | Solve a 3-term quadratic in $\tan x$, for $x$ | M1 | |
| | Obtain answer, e.g. $x = 31.7°$ | A1 | |
| | Obtain second answer, e.g. $x = 121.7°$, and no other in the interval | A1 | Ignore answers outside the given interval. |
| | | 6 | |

### 28. 9709/32/F/M/21 Q5

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | State $R = \sqrt{11}$ | B1 | |
| | Use trig formulae to find $\alpha$ | M1 | |
| | Obtain $\alpha = 37.09°$ | A1 | |
| | | 3 | |
| 5(b) | Evaluate $\sin^{-1}\left(\dfrac{1}{\sqrt{11}}\right)$ to at least 2 dp (17.5484°) | B1 FT | The FT is on $R$. |
| | Use correct method to find a value of $\theta$ in the interval | M1 | |
| | Obtain answer, e.g. 62.7° | A1 | |
| | Use a correct method to obtain a second answer | M1 | |
| | Obtain second answer, e.g. 170.2°, and no other in the interval | A1 | Ignore answers outside the given interval. |
| | | 5 | |

### 29. 9709/31/M/J/21 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Use correct trig expansions and obtain an equation in sin $x$ and cos $x$ | *M1 | |
| | Use correct exact trig ratios for $30°$ in *their* expansion | B1 FT | e.g. $\cos x\left(\frac{\sqrt{3}}{2}-1\right)=\sin x\left(\sqrt{3}-\frac{1}{2}\right)$ |
| | Obtain an equation in tan $x$ | DM1 | Allow if their error in line 1 was a sign error |
| | Obtain $\tan x = \dfrac{2-\sqrt{3}}{1-2\sqrt{3}}$ from correct working | A1 | AG |
| | | 4 | |
| 3(b) | Obtain answer in the given interval, e.g.$173.8°$ | B1 | Accept $174°$, $354°$ or better |
| | Obtain a second answer and no other in the given interval, e.g. $353.8°$ | B1 | Ignore answers outside the given interval. Treat answers in radians (3.03 and 6.17) as a misread. |
| | | 2 | |

### 30. 9709/31/M/J/21 Q4

| Question | Answer | Marks |
|---|---|---|
| 4(a) | Use correct double angle formula or *t*-substitution twice | M1 |
| | Obtain $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ from correct working | A1 |
| | | 2 |

### 31. 9709/32/M/J/21 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Express the LHS in terms of cos $2\theta$ and sin $2\theta$ | B1 | e.g. $\dfrac{1}{\sin 2\theta}-\dfrac{\cos 2\theta}{\sin 2\theta}$ |
| | Use correct double angle formulae to express the LHS in terms of cos $\theta$ and sin $\theta$ | M1 | e.g. $\dfrac{1-\left(1-2\sin^2\theta\right)}{2\sin\theta\cos\theta}$ |
| | Obtain tan $\theta$ from correct working | A1 | AG |
| | **Alternative method for Question 6(a)** | | |
| | Express the LHS in terms of sin $2\theta$ and tan $2\theta$ | B1 | |
| | Use correct double angle formulae to express the LHS in terms of cos $\theta$ and sin $\theta$ | M1 | e.g. $\dfrac{1}{2\sin\theta\cos\theta}-\dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{2\dfrac{\sin\theta}{\cos\theta}}\left(=\dfrac{4\sin^2\theta}{4\sin\theta\cos\theta}\right)$ |
| | Obtain tan $\theta$ from correct working | A1 | AG |
| | **Alternative method for Question 6(a)** | | |
| | Express the LHS in terms of sin $2\theta$ and tan $2\theta$ | B1 | |
| | Use correct $t$ substitution or rearrangement of sin $2\theta$ in terms of $\sec^2 2\theta$ and tan $\theta$ to express the LHS in terms of tan $\theta$. | M1 | $\left(\dfrac{\sec^2\theta}{2\tan\theta}-\dfrac{1-\tan^2\theta}{2\tan\theta}=\right)\dfrac{1+\tan^2}{2\tan}-\dfrac{1-\tan^2}{2\tan}$ |
| | Obtain tan $\theta$ from correct working | A1 | AG |
| | | 3 | |

## 32. 9709/32/M/J/21 Q10

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | State or imply $CD = 2r - 2r\cos x$ | B1 | |
| | Using correct formulae for area of sector and trapezium, or equivalent, form an equation in $r$ and $x$ | M1 | e.g. $2 \times \frac{1}{2} r^2 x = \frac{0.9}{2}(2r + 2r - 2r\cos x) r\sin x$ |
| | Obtain $x = 0.9(2 - \cos x)\sin x$ | A1 | AG, NFWW |
| | | 3 | |
| 10(b) | Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$ | M1 | Calculated for both values and correct for one value is sufficient for M1. Must be working in radians. |
| | Complete the argument correctly with correct values | A1 | Must have sufficient accuracy to support the answer e.g. $\begin{array}{ccc} 0.5 > 0.484 & 0.016 > 0 & 0.96... < 1 \\ \text{or} & \text{or} & \\ 0.7 < 0.716 & -0.016 < 0 & 1.02... > 1 \end{array}$ |
| | | 2 | |

## 33. 9709/33/M/J/21 Q5

| Question | Answer | Marks |
|---|---|---|
| 5(a) | Use double angle formula to express $\tan 4\theta$ in terms of $\tan 2\theta$ | M1 |
| | Use double angle formula to express result in terms of $\tan \theta$ | M1 |
| | Obtain a correct equation in $\tan \theta$ in any form | A1 |
| | Obtain the given answer | A1 |
| | | 4 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | Solve for $\tan \theta$ and obtain a value of $\theta$ | M1 | |
| | Obtain answer, e.g. 53.5° | A1 | |
| | Obtain second answer, e.g. 126.5° and no other in the interval | A1 | Ignore answers outside the given interval. Treat answers in radians as a misread. |
| | | 3 | |

## 34. 9709/31/O/N/21 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | State $R = \sqrt{34}$ | B1 | |
| | Use trig formulae to find $\alpha$ | M1 | $\tan\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{3}{\sqrt{34}}$ or $\cos\alpha = \frac{5}{\sqrt{34}}$ . |
| | Obtain $\alpha = 0.54$ | A1 | 30.96° scores M1A0. |
| | | 3 | |
| 2(b) | State greatest value 34 | B1 FT | *Their* $R^2$. |
| | State least value 0 | B1 | |
| | | 2 | |

## 35. 9709/31/O/N/21 Q5

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Use correct trig formulae and express equation in terms of $\tan\theta$ | M1 | |
| | Obtain a correct equation in $\tan\theta$ in any form | A1 | e.g. $\dfrac{1-\tan^2\theta}{2\tan\theta}+\dfrac{1}{\tan\theta}=2$ |
| | Reduce to $\tan^2\theta+4\tan\theta-3=0$, or 3-term equivalent | A1 | |
| | | 3 | |
| 5(b) | Solve a 3-term quadratic for $\tan\theta$ and calculate $\theta$ | M1 | $\left(\tan\theta=-2\pm\sqrt{7}\right)$ |
| | Obtain answer, e.g. 0.573 | A1 | Must be 3 d.p. |
| | Obtain second answer, e.g. 1.783 and no other | A1 | Ignore answers outside the given interval. Treat answers in degrees as a misread. $(32.9°,102.1°)$ |
| | | 3 | |

## 36. 9709/32/O/N/21 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$ | B1 | |
| | Substitute expansions in $\dfrac{1}{2}(\sin5x+\sin x)$, or equivalent | M1 | |
| | Simplify and obtain $\dfrac{1}{2}(\sin5x+\sin x)=\sin3x\cos2x$ | A1 | Obtain the **given identity** correctly. |
| | | 3 | |

## 37. 9709/32/O/N/21 Q8

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Expand the square and equate to 1 | B1 | |
| | Use correct double angle formula | M1 | Need to see $\dfrac{4}{2}$ or $\sin2\theta=2\sin\theta\cos\theta$ stated. |
| | Obtain $\cos^4\theta+\sin^4\theta=1-\tfrac{1}{2}\sin^2 2\theta$ | A1 | Obtain the **given result** correctly. |
| | | 3 | |
| 8(b) | Use the identity and carry out a method for finding a root | M1 | $\left(1-\tfrac{1}{2}\sin^2 2\theta=\tfrac{5}{9}\right)$ |
| | Obtain answer 35.3° | A1 | Must be correct if overspecified: 35.264... |
| | Obtain a second answer, e.g. 54.7° | A1 FT | $\left[\text{e.g } 90°-\textit{their } 35.3°\right]$ Do not FT if mixing degrees and radians. |
| | Obtain the remaining answers, e.g. 144.7° and 125.3° and no others in the given interval | A1 FT | $\left[\text{e.g.} 180°-.. \text{ and } 180°-..\right]$ Ignore answers outside the given interval. Treat answers in radians as a misread. $(0.615, 0.955, 2.19, 2.53)$ Do not FT if mixing degrees and radians. |
| | | 4 | |

### 38. 9709/33/O/N/21 Q5

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | Use double angle formula and obtain an equation in $\sin\theta$ | M1 | |
| | Reduce to $6\sin^2\theta + \sin\theta - 5 = 0$, or 3-term equivalent | A1 | |
| | Solve a 3-term quadratic in $\sin\theta$ and calculate $\theta$ | M1 | |
| | Obtain answer, e.g. 56.4° | A1 | |
| | Obtain second and third answers, e.g.123.6° and 270° and no others in the given interval | A1 | Ignore answers outside the interval. Treat answers in radians as a misread. |
| | | 5 | |

### 39. 9709/33/O/N/21 Q6

| Question | Answer | Marks |
|---|---|---|
| 6(a) | Use $\cos(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$ | M1 |
| | Collect terms and reach $2\cos x + \sqrt{3}\,\sin x$ | A1 |
| | State $R = \sqrt{7}$ | A1 |
| | Use trig formula to find $\alpha$ | M1 |
| | Obtain $\alpha = 40.89°$ | A1 |
| | | 5 |
| 6(b) | Use correct method to find $x$ | M1 |
| | Obtain answer $x = 220.9°$ | A1 |
| | | 2 |

### 40. 9709/32/F/M/22

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\alpha$ and $\tan\beta$ | M1 | $\dfrac{\tan + \tan}{1\ \tan\tan} = 2$ |
| | Substitute throughout for $\tan\alpha$ or for $\tan\beta$ | M1 | $\dfrac{3\tan + \tan}{1\ 3\tan^2} = 2$ |
| | Obtain $3\tan^2\beta + 2\tan\beta - 1 = 0$ or $\tan^2\alpha + 2\tan\alpha - 3 = 0$ | A1 | OE e.g. $6\tan^2\beta + 4\tan\beta - 2 = 0$ or $\dfrac{2}{3}\tan^2\alpha + \dfrac{4}{3}\tan\alpha - 2 = 0$ |
| | Solve a 3-term quadratic and find an angle | M1 | |
| | Obtain answer $\alpha = 45°$, $\beta = 18.4°$ | A1 | $\dfrac{\pi}{4}$ or 0.785, 0.322 |
| | Obtain answer $\alpha = 108.4°$, $\beta = 135°$ | A1 | 1.89 , $\dfrac{3\pi}{4}$ or 2.36. Answer in radians, max. A1A0 or vice versa. Ignore answers outside [0°, 180°] |
| | | 6 | SC: If A0A0 allow **SC B1** for both $\alpha$'s or both $\beta$'s |

### 41. 9709/31/M/J/22 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use correct trigonometric formulae to form an equation in $\tan x$ | *M1 | e.g. $\dfrac{1-\tan^2 x}{\tan x}+\dfrac{3}{\tan x}=5$ |
| | Obtain a correct linear equation in any form | A1 | $1-\tan^2 x+3=5\tan x$ |
| | Reduce equation to a 3-term quadratic | A1 | $\tan^2 x+5\tan x-4=0$, or 3-term equivalent |
| | Solve a 3-term quadratic in $\tan x$ and obtain a value of $x$ | DM1 | |
| | Obtain answer, e.g. $x=35.1°$ | A1 | |
| | Obtain second answer, e.g. $x=99.9°$, and no other in $(0°,180°)$ | A1 | Ignore answers outside $(0°,180°)$. Treat answers in radians $(0.612,1.74)$ as a misread. |
| | **Alternative method for question 3** | | |
| | Use correct formulae for $\sin 2x$ and $\cos 2x$ to form an equation in $\sin x$ and $\cos x$ | *M1 | |
| | Obtain $4\dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}=5$ | A1 | |
| | Reduce equation to a 3-term quadratic | A1 | $\tan^2 x+5\tan x-4=0$, or 3-term equivalent |
| | Solve a 3-term quadratic in $\tan x$ and obtain a value of $x$ | DM1 | |
| | Obtain answer, e.g. $x=35.1°$ | A1 | |
| | Obtain second answer, e.g. $x=99.9°$, and no other in $(0°,180°)$ | A1 | Ignore answers outside $(0°,180°)$. Treat answers in radians $(0.612,1.74)$ as a misread. |
| | | 6 | |

### 42. 9709/32/M/J/22 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use correct double-angle formula to obtain an equation in $\cos\theta$ | M1 | e.g. $3(2\cos^2\theta-1)=3\cos\theta+2$ |
| | Obtain $6\cos^2\theta-3\cos\theta-5=0$, or 3-term equivalent | A1 | M1 A0 is scored if they use any correct formula for $\cos 2\theta$ and make a subsequent error. |
| | Solve a 3-term quadratic in $\cos\theta$ for $\theta$ | M1 | As far as $\theta=\cos^{-1}\left(\dfrac{3-\sqrt{129}}{12}\right)$ if quadratic correct. |
| | Obtain a correct answer, e.g. $134.1°$ | A1 | Accept greater accuracy e.g. 134.1456, 225.8544. |
| | Obtain a second answer, e.g. $225.9°$ and no other in $[0°,360°]$ | A1 FT | Treat answers in radians $(2.34$ and $3.94)$ as a misread. Ignore answers outside $[0°,360°]$. The FT is for 360° minus the first answer. |
| | | | **Special Ruling**: If they have an **incorrect** quadratic that leads legitimately to 4 solutions for $\theta$, allow FT for 360° minus an answer in $(0°,180°)$. More than 4 solutions is maximum M1 A0 M1 A0 A0. If *their* equation should have 4 solutions and the candidate only gives 3 solutions then M1 A0 M1 A0 A0. Mis-read leading to a quadratic with 4 solutions could score maximum M1 A0 M1 A1 A1 or M1 A0 M1 A1 A0 if extra/missing solution. |
| | | 5 | |

### 43. 9709/33/M/J/22 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use correct $\cos(A-B)$ formula to obtain an equation in $\cos\theta$ and $\sin\theta$ | B1 | $\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$ |
| | Use trigonometric formula and substitute values for cos 60 and sin 60 to obtain an equation in $\tan\theta$ (or $\cos\theta$ or $\sin\theta$) | M1 | Allow $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ interchanged. $\frac{1}{2}+\frac{\sqrt{3}}{2}\tan\theta = 3\tan\theta$ $\frac{1}{4}\cos^2\theta = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)(1-\cos^2\theta)$ $\frac{1}{4}(1-\sin^2\theta) = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)\sin^2\theta$ |
| | Obtain $\tan\theta = \dfrac{1}{6-\sqrt{3}}$ or $\tan\theta = \dfrac{6+\sqrt{3}}{33}$ or 0.2343, $\cos\theta = \dfrac{3\frac{\sqrt{3}}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.9736 or $\sin\theta = \dfrac{\frac{1}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.2281 | A1 | OE |
| | Obtain answer, e.g. $\theta = 13.2°$ | A1 | May be more accurate, allow value rounding to $13.2°$. $\theta = 13.1867°$. |
| | Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval | A1 FT | May be more accurate. Allow value rounding to $193.2°$. FT is on previous value of $\theta$, must have scored M1. Note if $\theta$ is negative (e.g. $-13.2$): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717. |

### 44. 9709/31/O/N/22 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Use correct $\tan(A+B)$ formula and obtain an equation in $\tan x$ or an equation in $\cos x$ and $\sin x$ | M1 | e.g. $\dfrac{\tan x + \tan 45°}{1-\tan x\tan 45°} = \dfrac{2}{\tan x}$ Allow if 2 in denominator or $\dfrac{\sin x\cos 45° + \cos x\sin 45°}{\cos x\cos 45° - \sin x\sin 45°} = \dfrac{2\cos x}{\sin x}$. |
| | Obtain correct 3 term equation $\tan^2 x + 3\tan x - 2 = 0$, or equivalent | A1 | or $3\sin x\cos x = 2\cos^2 x - \sin^2 x$ |
| | Solve a 3-term quadratic in $\tan x$ and obtain a value for $x$ | M1 | |
| | Obtain answer, e.g. $29.3°$ | A1 | 29.316... |
| | Obtain second answer, e.g. $105.7°$ and no other | A1 | 105.583.... Ignore answers outside the given interval. Treat answers in radians as a misread. |
| | | 5 | |

## 45. 9709/31/O/N/22 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$ | **B1** | |
| | Express $\cos 2\theta$ in terms of $\cos\theta$ and/or $\sin\theta$ | **B1** | Anywhere |
| | Expand to obtain a correct expression in terms of $\cos\theta$ | **B1** | e.g. $2\left(2\cos^2\theta - 1\right)^2 - 1 + 4\left(2\cos^2\theta - 1\right) + 3$ |
| | Reduce correctly to $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$ | **B1** | AG |
| | | **4** | |
| 6(b) | Use the identity and carry out method to calculate a root | **M1** | $8\cos^4\theta - 3 = 4$ |
| | Obtain answer, e.g. $14.7°$ | **A1** | |
| | Obtain second answer, e.g. $165.3°$, and no other in the given interval | **A1 FT** | Ignore answers outside the given interval. Treat answers in radians as a misread. |
| | | **3** | |

## 46. 9709/32/O/N/22 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use correct product rule on given expression | **\*M1** | |
| | Obtain correct derivative in any form | **A1** | e.g. $\cos x \sin 2x + 2\sin x \cos 2x$ |
| | Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$ | **\*M1** | |
| | Equate derivative to zero and obtain an equation in one trig variable | **DM1** | dependent on the 2 previous M Marks. |
| | Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$ | **A1** | OE |
| | Solve and obtain $x = 0.955$ | **A1** | 3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of $y$. |
| | **Alternative method for the first three marks** | | |
| | Use correct double angle formula to obtain $y = 2\cos x - 2\cos^3 x$ | **\*M1** | or $y = 2\sin^2 x \cos x$ |
| | Use chain rule and / or product rule | **\*M1** | |
| | Obtain derivative $y' = -2\sin x + 6\sin x \cos^2 x$ | **A1** | $y' = -2\sin^3 x + 4\sin x \cos^2 x$ |
| | **Alternative method for the second and third M marks** | | |
| | Equate derivative to zero and obtain an equation in $\tan x$ and $\tan 2x$ | **\*M1** | |
| | Use correct double angle formula to obtain an equation in $\tan x$ | **DM1** | |
| | | **6** | |

### 47. 9709/32/O/N/22 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | State $R = \sqrt{17}$ | **B1** | Allow if working from an incorrect expansion but not from decimals. |
| | Use correct trig formulae to find $\alpha$ (Correct expansion and correct expression for trig ratio for $\alpha$ ) | **M1** | NB: $\cos\alpha = 4$ and $\sin\alpha = 1$ scores M0A0. M0 for incorrect expansion of $\cos(x - \alpha)$ M1 for correct expression for trig ratio for $\alpha$ and no errors seen. |
| | Obtain $\alpha = 14.04°$ | **A1** | 2 d.p. required Allow M1A1 for correct answer with no working shown. Correct answer from incorrect working (e.g. $\tan^{-1}\left(-\frac{1}{4}\right)$) is awarded M0A0. $180° - \tan^{-1}\left(-\frac{1}{4}\right)$ is awarded M1 |
| | | **3** | |
| 4(b) | Evaluate $\cos^{-1}\left(\dfrac{3}{\sqrt{17}}\right)$ to at least 1 d.p. (43.31 38...°) | **B1 FT** | FT *their R*. Accept awrt $43.3°$ or awrt $316.7°$ Can be implied by subsequent working. |
| | Use correct method to find a value of $x$ in the interval | **M1** | Must be working with $2x$ and *their* $\alpha$ . |
| | Obtain answer, e.g. 14.6 ° | **A1** | Accept overspecified answers but they need to be correct. (14.6388.... and 151.3249...). |
| | Use a correct method to find a second answer in the interval | **M1** | Must be working with $2x$, *their* $\alpha$ and $360° -$ *their* 43.3 . |
| | Obtain second answer in the interval, e.g. 151.3 °, and no other in the interval | **A1** | Ignore answers outside the given interval. Treat answers in radians (0.255… and 2.64…) as a misread. |
| | | **5** | |

### 48. 9709/33/O/N/22 Q7

| Question | Answer | Marks |
|---|---|---|
| 7(a) | Rearrange and obtain $4\cos x - \sin x = \sqrt{5}$ | **B1** |
| | State $R = \sqrt{17}$ | **B1** |
| | Use trig formulae to find $\alpha$ | **M1** |
| | Obtain $\alpha = 14.04°$ | **A1** |
| | | **4** |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | Evaluate $\cos^{-1}\left(\dfrac{\sqrt{5}}{\sqrt{17}}\right)$ | **B1 FT** | FT *their R*. |
| | Carry out a correct method to find a value of $x$ in the given interval | **M1** | |
| | Obtain answer, e.g. 21.6 ° | **A1** | |
| | Obtain a second answer, e.g. 144.4 ° and no other in the interval | **A1** | Treat answers in radians as a misread. Ignore answers outside the given interval. |
| | | **4** | |

## 49. 9709/32/F/M/23 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | State $R = 13$ | B1 | Allow if $\sqrt{(12^2 + (-5)^2)}$ seen. |
| | Use correct trig formulae to find $\alpha = \tan^{-1}(\pm5/12) = \cos^{-1}(\pm12/13) = \sin^{-1}(\pm5/13)$ | M1 | $\cos(\alpha) = 12$ and $\sin(\alpha) = 5$ M0<br>However, $\sin(\alpha)/\cos(\alpha) = 5/12$ or $-5/12$ with no error seen, or $\tan(\alpha) = 5/12$ or $-5/12$ quoted then allow. |
| | Obtain $\alpha = 0.395$ | A1 | CWO If negative sign seen when finding $R$ then A0 here.<br>If degrees 22.6 A0 MR.<br>Only penalise degrees once in (a) and (b).<br>Note $\alpha = 0.39479\ldots$ |
| | | 3 | |
| 6(b) | $\cos^{-1}\left(\dfrac{6}{R}\right)$ | B1FT | SOI<br>1.0910... FT *their* incorrect $R$. |
| | Use correct method to find a value of $2x$ in the interval | M1 | $2x = \cos^{-1}\left(\dfrac{6}{R}\right) + \alpha$ or $2\pi - \cos^{-1}\left(\dfrac{6}{R}\right) + \alpha.$<br>Allow if $\cos(2x + 0.395)$ seen |
| | Obtain answer, e.g. $x = 0.743$ or $0.742$ | A1 | 42.5 or 42.6 degrees. |
| | Obtain second answer, e.g. $x = 2.79$ and no others in the interval | A1 | 159.8, 159.9 or 160.0 degrees all possible depending whether using 3 dp or 4 dp. |
| | | 4 | |

## 50. 9709/32/F/M/23 Q7

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | State or imply area of major sector $= \dfrac{1}{2}r^2(2\pi - x)$ | B1 | OE |
| | State or imply area of shaded segment $= \dfrac{1}{2}r^2x - \dfrac{1}{2}r^2\sin x$ | B1 | OE<br>$r^2\sin(x/2)\cos(x/2)$ B0 until changed to $(1/2)r^2\sin x$. |
| | State $\dfrac{1}{2}r^2(2\pi - x) = 3\left(\dfrac{1}{2}r^2x - \dfrac{1}{2}r^2\sin x\right)$ | M1 | OE<br>Area of major sector = 3 times (area of minor sector – area of triangle).<br>Allow $r^2\sin(x/2)\cos(x/2)$. |
| | Obtain the given answer $x = \dfrac{3}{4}\sin x + \dfrac{1}{2}\pi$ after full and correct working | A1 | AG Allow rectified slip if before penultimate line. |
| | | 4 | |
| 7(b) | Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.5$ | M1 | $x = 2$          $x = 2.5$<br>$(3/4)\sin x + (1/2)\pi$ 2.2(5277)    2.0(197)<br>      $2 < 2.2$ or 2.3       $2.5 > 2.0$<br>$x - (3/4)\sin x - (1/2)\pi$<br>     $-0.2(5277) < 0$        $+0.4(803) > 0$<br>                    or change of sign<br>Attempt both values and one correct for M1. |
| | Complete the argument correctly with correct calculated values | A1 | Degrees award 0/2 |
| | | 2 | |

## 51. 9709/31/M/J/23 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Use correct double angle formulae | M1 | e.g. $2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta = 2\sin^2\theta$ |
| | Obtain $\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$ from **full and correct** working | A1 | AG Check conclusion is complete and matches the working. |
| | | 2 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | Factorise to obtain $(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$ | B1 | OE |
| | Solve a quadratic in $\sin\theta$ and $\cos\theta$ to obtain a value for $\theta$. | M1 | $\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$. |
| | Obtain one correct value e.g. $45°$ | A1 | |
| | Obtain a second correct value e.g. $161.6°$ and no others in the interval | A1 | Mark answers in radians (0.785 and 2.82) as a misread. Accept awrt 161.6 . |
| | **Alternative Method 1** | | |
| | Obtain $3\tan^2\theta - 2\tan\theta - 1 = 0$ | B1 | |
| | Solve a 3 term quadratic in $\tan\theta$ to obtain a value for $\theta$. | M1 | $\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$. |
| | Obtain one correct value e.g. $45°$ | A1 | |
| | Obtain a second correct value e.g. $161.6°$ and no others in the interval | A1 | Mark answers in radians (0.785 and 2.82) as a misread. |
| | **Alternative Method 2** | | |
| | Obtain $(\cos\theta + \sin\theta)^2 = (2\sin\theta)^2$ | B1 | |
| | Solve to obtain a value for $\theta$. | M1 | $\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$. |
| | Obtain one correct value e.g. $45°$ | A1 | |
| | Obtain a second correct value e.g. $161.6°$ and no others in the interval | A1 | Mark answers in radians (0.785 and 2.82) as a misread. |
| | | 4 | |

## 52. 9709/32/M/J/23 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only | *M1 | e.g. $2\left(2\cos^2\left(\frac{x}{2}\right) - 1\right) - \cos\left(\frac{x}{2}\right) = 1$. |
| | Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$, | A1 | e.g. $4\cos^2\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right) - 3 = 0$. Allow $4\cos^2 u - \cos u - 3 = 0$. Condone $\frac{x}{2} = x$. |
| | Obtain $\cos\left(\frac{x}{2}\right) = -\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right) = 1$ | A1 | Allow answer in $u$ e.g. $(4\cos u + 3)(\cos u - 1)$ and condone $\frac{x}{2} = x$. |
| | Solve for the **original** $x$ | DM1 | Must see evidence of doubling, not halving. |
| | Obtain $x = 0$ and 4.84 and no others in the interval | A1 | Ignore any answers outside interval. Accept AWRT 4.84. Accept $1.54\pi$. Must be in radians. 277.2 indicates M1 but is A0. |
| | **Alternative Method for Question 4** | | |
| | Use correct double angle formula to obtain an equation in $\cos x$ only | *M1 | e.g. $2\cos x - 1 = \sqrt{\dfrac{\cos x + 1}{2}}$. |
| | Obtain a 3 term quadratic in $\cos x$, | A1 | e.g. $8\cos^2 x - 9\cos x + 1 = 0$. |
| | Obtain $\cos x = \frac{1}{8}$ and $\cos x = 1$ | A1 | |
| | Solve for $x$ | DM1 | |
| | Obtain answers $x = 0$ and 4.84 and no others in the interval | A1 | Ignore any answers outside interval. Accept AWRT 4.84. Must be in radians. 277.2 is A0. |
| | | 5 | |

### 53. 9709/32/M/J/23 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Calculate the values of a relevant expression or pair of expressions at $x$ = 0.5 and $x$ = 1 | M1 | Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using g($x$) – f($x$), there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient. |
| | Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required. | A1 | e.g. 3.92 > 1.5, 1.83 < 3 or 2.42 > 0, –1.17 < 0. |
| | | 2 | |

### 54. 9709/33/M/J/23 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Expand $\cos(x-60°)$ correctly and evaluate $3\cos x + 2\cos(x-60°)$ to obtain $4\cos x + \sqrt{3}\sin x$ or unsimplified coefficients | B1 | Need to see $A\cos x + B\sin x$ with $A$ and $B$ correct $A$ may be 4 or $3 + 2\cos 60$ and $B$ may be $\sqrt{3}$ or $2\sin 60$. |
| | State $R = \sqrt{19}$ [$R\cos\alpha = 4$ $R\sin\alpha = \sqrt{3}$] | B1 FT | Follow through *their* 4 and $\sqrt{3}$. If coefficients are 3 and 2 then B0. $R = \sqrt{19}$ from $R = 4.36$ B0 but 4.36 seen after $\sqrt{19}$ ISW. |
| | Use correct trig formulae for their expansion to find $\alpha$ e.g. $\alpha = \tan^{-1}\dfrac{\sqrt{3}}{4}$ or $\cos^{-1}\dfrac{4}{\sqrt{19}}$ or $\sin^{-1}\dfrac{\sqrt{3}}{\sqrt{19}}$ | M1 | If $\sin\alpha = \sqrt{3}$ $\cos\alpha = 4$ seen then M0 A0. If <u>tan</u> $\alpha = 23.41°$ M0 A0 but can recover if $\alpha = 23.41°$ seen later. $\alpha = \tan^{-1}\dfrac{2}{3}$ M1 ($\alpha = 33.69°$) but $\alpha = \tan^{-1}\dfrac{3}{2}$ M0 |
| | Obtain $\alpha = 23.41°$ | A1 | Allow if $x$ instead of $\alpha$. |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\cos^{-1}\left(\dfrac{2.5}{R}\right)$ | B1 FT | SOI [55.0°]. Follow through *their* $\sqrt{19}$. |
| | Use a correct method to find a value of $2\theta$ (not $x$) in the interval. Allow sign error in moving $\alpha$ to right side | M1 | $2\theta = \cos^{-1}\left(\dfrac{2.5}{R}\right) + 23.41°$ or $2\theta = 360° - \cos^{-1}\left(\dfrac{2.5}{R}\right) + 23.41°$ with $R$ substituted. |
| | Obtain one correct answer e.g. 39.2° | A1 | If working for **M1** not seen then **M1** implied by 39.2° or 164.2° Must be at least 1d.p. |
| | Obtain second correct answer e.g. 164.2° and no others in the interval | A1 | Must be at least 1d.p. Ignore answers outside the given interval. |
| | | 4 | |

## 55. 9709/31/O/N/23 Q5

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Use correct trig formulae and obtain an equation in sin $x$ and cos $x$ | *M1 | Allow one sign error. |
| | Obtain a correct equation in any form | A1 | e.g $2\cos x\sin\frac{\pi}{6} = -2\sin x\sin\frac{\pi}{3}$. |
| | Substitute exact trig ratios and obtain an expression for tan $x$ | DM1 | Allow one sign error. |
| | Obtain answer $\tan x = -\dfrac{1}{\sqrt{3}}$ | A1 | Or exact equivalent. |
| | | 4 | |
| 5(b) | Obtain answer, e.g. $x = \dfrac{5\pi}{6}$ | B1 | |
| | Obtain second answer, e.g. $x = \dfrac{11\pi}{6}$ and no others in the interval | B1FT | FT first answer $+ \pi$ (provided $0 \leqslant$ first answer $\leqslant \pi$). Or FT first answer $- \pi$ (provided $\pi \leqslant$ first answer $\leqslant 2\pi$). Ignore any answers outside interval. |
| | | 2 | |

## 56. 9709/32/O/N/23 Q7

| Question | Answer | Marks |
|---|---|---|
| 7(a) | Use correct expansion for $\cos(2\theta + \theta)$ | *M1 |
| | Use correct double angle formulae to express $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$ | DM1 |
| | Show sufficient working to confirm $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$ | A1 |
| | | 3 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | Use the identity and correct double angle formula to obtain an equation in $\cos\theta$ only. Must come from using all three terms in the given equation. | *M1 | e.g. $4\cos^3\theta - 3\cos\theta + \cos\theta(2\cos^2\theta - 1) = \cos^2\theta$ $6\cos^3\theta - \cos^2\theta - 4\cos\theta = 0$ or $6\cos^2\theta - \cos\theta - 4 = 0$. |
| | Obtain $\theta = 90°$ | B1 | Allow if $\cos\theta$ obtained correctly as a factor of *their* expression (even if there is an error in the quadratic factor). Can follow M0. |
| | Solve a 3-term quadratic in $\cos\theta$ to obtain a value of $\theta$ | DM1 | |
| | Obtain one value e.g. 25.3° | A1 | Accept awrt 25.3°. |
| | Obtain a second value e.g. 137.5° and no extras in range | A1 | Accept awrt 137.5°. Ignore values outside the range. Mark solutions in radians as a misread $(0.442, 1.57, 2.40)$. |
| | | 5 | |

### 57. 9709/33/O/N/23 Q6

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Use correct Pythagoras $\cot^2\theta = \csc^2\theta - 1$ or $\cot^2\theta = 1/\sin^2\theta - 1$ or $\cot^2\theta = \cos^2\theta/\sin^2\theta$ and then $\cos^2\theta = 1 - \sin^2\theta$, together with double angle formula $\cos 2\theta = 1 - 2\sin^2\theta$, to obtain an equation in $\sin\theta$ or $\sin\theta$ and $\csc^2\theta$ | M1 | If consistent omission of brackets, e.g. $(\sin\theta)^2$ written as $\sin\theta^2$ then **SC B1** in place of M1A1. |
|  | Obtain a correct equation in $\sin\theta$ in any form | A1 | e.g. $1/\sin^2\theta - 1 + 2(1 - 2\sin^2\theta) = 4$ or $\dfrac{1-\sin^2}{\sin^2} + 2(1 - 2\sin^2) = 4$. If $\dfrac{\cos^2}{\sin^2} + 2(1 - 2\sin^2) = 4$ then e.g. $1 - \sin^2 + 2(1 - 2\sin^2)\sin^2 = 4$. (missing $\sin^2$ on right) allow M1A1A0. |
|  | Reduce to the given answer of $4\sin^4\theta + 3\sin^2\theta - 1 = 0$ correctly | A1 | AG Must follow from a horizontal equation (no denominators). If $s = \sin\theta$ used and defined, allow all marks. If not defined, award M1A1A0. |
|  |  | 3 |  |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | Solve the given quadratic to obtain a value for $\theta$ | M1 | $(4\sin^2\theta - 1)(\sin^2\theta + 1) = 0$ and solve for $\theta$. <br><br> Incorrect sign in solution of quadratic seen, e.g. $(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$ then M0 A0 A0 but if only see $(4\sin^2\theta - 1) = 0$ and nothing incorrect seen allow 3/3. |
|  | Obtain answer, e.g. $\theta = 30°$ | A1 | $\pi/6$ award A0 |
|  | Obtain three further answers, e.g. $\theta = 150°, 210°$ and $330°$ and no others in the interval | A1 | Ignore any answers outside interval. $5\pi/6$ $7\pi/6$ $11\pi/6$ award A1. |
|  |  | 3 |  |

### 58. 9709/32/F/M/24

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Use the correct expansion of $\cos(x + \frac{1}{4}\pi)$ to obtain $\sin x + 2\cos x$ | B1 | $3\sin x + 2\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\cos x - \dfrac{1}{\sqrt{2}}\sin x\right)$. |
|  | State $R = \sqrt{5}$ | B1 FT | ISW FT *their* $a\sin x + b\cos x$ provided this expression obtained by correct method. |
|  | Use correct trig formulae to find $\alpha$ | M1 | $\alpha = \tan^{-1}(b/a)$ from *their* $a\sin x + b\cos x$ or $\sin^{-1}$ or $\cos^{-1}$ provided this expression obtained by correct method. NB If $\cos\alpha = 1$ and $\sin\alpha = 2$ then **M0 A0**. |
|  | Obtain $\alpha = 1.107$ | A1 | 3 d.p. CAO Treat answer in degrees as a misread $(63.435°)$. |
|  |  | 4 |  |
| 8(b) | $\sin^{-1}\left(\dfrac{1.5}{R}\right)$ | B1 FT | Follow *their R*. |
|  | Use a correct method to obtain an un-simplified value of $\theta$ with *their* $\alpha$ | M1 | $2\left(\sin^{-1}\left(\dfrac{1.5}{R}\right) - \alpha\right)$ or $2\left(\pi - \sin^{-1}\left(\dfrac{1.5}{R}\right) - \alpha\right)$. |
|  | Obtain one correct answer e.g. $-0.74$ in the interval | A1 |  |
|  | Obtain second correct answer e.g. $2.60$ $(2.5986)$ or $4\pi - 0.74 = 11.8$ or $2.60 - 4\pi = -9.97$ in the interval | A1 | If uses $1.11°$ withhold first accuracy mark gained, but allow rest of accuracy marks. Allow $2.6(0)$. |
|  | Obtain two more correct answers e.g. $-9.97$ and $11.8$ and no others in the interval | A1 | Ignore answers outside the interval. Treat answers in degrees as a misread. $(-571.1°, -42.6°, 148.9°, 677.2°)$. |
|  |  | 5 |  |