1. 9709/32/F/M/18 Q4

| Question | Answer | Marks |
|----------|--------|-------|
| 4(i) | State or imply $n \ln y = \ln A + 3 \ln x$ | B1 |
| | State that the graph of ln $y$ against ln $x$ has an equation which is *linear* in ln $y$ and ln $x$, or has equation of the form $nY = \ln A + 3X$, where $Y = \ln y$ and $X = \ln x$, and is thus a straight line. | B1 |
| | | 2 |
| 4(ii) | Substitute $x$- and $y$-values in $n \ln y = \ln A + 3 \ln x$ or in the given equation and solve for one of the constants | M1 |
| | Obtain a correct constant, e.g. $n = 1.70$ | A1 |
| | Solve for a second constant | M1 |
| | Obtain the other constant, e.g. $A = 2.90$ | A1 |
| | | 4 |

2. 9709/31/M/J/18 Q1

| Question | Answer | Marks |
|----------|--------|-------|
| 1 | Use law for the logarithm of a product, quotient or power | M1 |
| | Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$ | A1 |
| | Solve for $x$ | M1 |
| | Obtain answer $x = 1.52$ only | A1 |
| | | 4 |

### 3. 9709/32/M/J/18 Q1

| Question | Answer | | Marks | Guidance |
|---|---|---|---|---|
| 1 | *EITHER*: | State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm2^x$ | M1 | $8\left(2^x\right)^2-18\left(2^x\right)+9=0$ |
| | | Obtain $2^x=\dfrac{3}{2}$ and $2^x=\dfrac{3}{4}$ or equivalent | A1 | |
| | *OR*: | Obtain $2^x=\dfrac{3}{2}$ by solving an equation | B1 | |
| | | Obtain $2^x=\dfrac{3}{4}$ by solving an equation | B1 | |
| | | Use correct method for solving an equation of the form $2^x=a$, where $a>0$ | M1 | |
| | | Obtain **final** answers $x=0.585$ and $x=-0.415$ only | A1 | The question requires 3 s.f. Do not ISW if they go on to reject one value |
| | | | 4 | |

### 4. 9709/33/M/J/18 Q1

| Question | Answer | Marks |
|---|---|---|
| 1 | Obtain a correct unsimplified version of the $x$ or $x^2$ term of the expansion of $\left(4-3x\right)^{-\frac{1}{2}}$ or $\left(1-\dfrac{3}{4}x\right)^{-\frac{1}{2}}$ | M1 |
| | State correct first term 2 | B1 |
| | Obtain the next two terms $\dfrac{3}{4}x+\dfrac{27}{64}x^2$ | A1 + A1 |
| | Total: | 4 |

### 5. 9709/31/O/N/18 Q2

| Question | Answer | Marks |
|---|---|---|
| 2 | Rearrange the equation in the form $ae^{2x}=b$ or $ae^x=be^{-x}$ | M1 |
| | Obtain correct equation in either form with $a=2$ and $b=5$ | A1 |
| | Use correct method to solve for $x$ | M1 |
| | Obtain answer $x=0.46$ | A1 |
| | | 4 |

### 6. 9709/32/O/N/18 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Substitute and obtain 3-term quadratic $3u^2 + 4u - 1 = 0$, or equivalent | B1 | e.g. $3(e^x)^2 + 4e^x - 1 = 0$ |
| | Solve a 3 term quadratic for $u$ | M1 | Must be an equation with real roots |
| | Obtain root $(\sqrt{7} - 2)/3$, or decimal in [0.21, 0.22] | A1 | Or equivalent. Ignore second root (even if incorrect) |
| | Use correct method for finding $x$ **from a positive value of** $e^x$ | M1 | Must see some indication of method: use of $x = \ln u$ |
| | Obtain answer $x = -1.536$ only | A1 | CAO. Must be 3 dp |
| | | 5 | |

### 7. 9709/33/O/N/18 Q2

| Question | Answer | Marks |
|---|---|---|
| 2 | Rearrange the equation in the form $ae^{2x} = b$ or $ae^x = be^{-x}$ | M1 |
| | Obtain correct equation in either form with $a = 2$ and $b = 5$ | A1 |
| | Use correct method to solve for $x$ | M1 |
| | Obtain answer $x = 0.46$ | A1 |
| | | 4 |

### 8. 9709/32/F/M/19 Q1

| Question | Answer | Marks |
|---|---|---|
| 1(i) | Use law for the logarithm of a product or quotient | M1 |
| | Use $\log_{10} 100 = 2$ or $10^2 = 100$ | M1 |
| | Obtain $x^2 - 4x - 100 = 0$, or equivalent | A1 |
| | | 3 |
| 1(ii) | Solve a 3-term quadratic equation | M1 |
| | Obtain answer 12.2 only | A1 |
| | | 2 |

## 9. 9709/31/M/J/19 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use law for the logarithm of a product, quotient or power | M1 | Condone $\ln\dfrac{x}{x-1}$ for M1 |
| | Obtain a correct equation free of logarithms | A1 | e.g. $(2x-3)(x-1)=x^2$ or $x^2-5x+3=0$ |
| | Solve a 3-term quadratic obtaining at least one root | M1 | Must see working if using an incorrect quadratic $\left(\dfrac{5\pm\sqrt{13}}{2}\right)$ |
| | Obtain answer $x = 4.30$ only | A1 | Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0 |
| | | 4 | |

## 10. 9709/32/M/J/19 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | State or imply $u^2 - u - 12 (= 0)$, or equivalent in $3^x$ | B1 | Need to be convinced they know $3^{2x} = \left(3^x\right)^2$ |
| | Solve for $u$, or for $3^x$, and obtain root 4 | B1 | |
| | Use a correct method to solve an equation of the form $3^x = a$ where a >0 | M1 | Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x\ln 3 = \ln a$, $x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working |
| | Obtain final answer $x = 1.26$ **only** | A1 | The Q asks for 2 dp |
| | | 4 | |

## 11. 9709/33/M/J/19 Q1

| Question | Answer | Mar |
|---|---|---|
| 1 | Use law of the logarithm of a product or quotient | |
| | Use law of the logarithm of power **twice** | |
| | Obtain a correct linear equation in $x$, e.g. $(3-2x)\ln 5 = \ln 4 + x\ln 7$ | |
| | Obtain answer $x = 0.666$ | |
| | | |

## 12. 9709/31/O/N/19 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | State $1 + e^{2y} = e^x$ | B1 | |
| | Make $y$ the subject | M1 | Rearrange to $e^{2y} = \ldots$ and use logs |
| | Obtain answer $y = \dfrac{1}{2}\ln\left(e^x - 1\right)$ | A1 | OE |
| | | 3 | |

## 13. 9709/32/O/N/19 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Remove logarithms and state $4 - 3^x = e^{1.2}$, or equivalent | B1 | Accept $4 - 3^x = 3.32(01169.....)$ 3 s.f. or better |
| | Use correct method to solve an equation of the form $3^x = a$, where $a > 0$. | M1 | $\left(3^x = 0.67988..\right)$ <br> Complete method to $x = \ldots$ <br> If using $\log_3$ the subscript can be implied |
| | Obtain answer $x = -0.351$ only | A1 | CAO must be to 3 d.p. |
| | | 3 | |

## 14. 9709/33/O/N/19 Q3

| Question | Answer | Marks |
|---|---|---|
| 3 | Reduce the equation to a horizontal equation in $3^{3x}$, $3^{3x+1}$ or $27^x$ | M1 |
| | Simplify and reach $3(3^{3x}) = 5$, $3\left(27^x\right) = 5$, or equivalent | A1 |
| | Use correct method for finding $x$ from a positive value of $3^{3x}$, $3^{3x+1}$ or $27^x$ | M1 |
| | Obtain answer $x = 0.155$ | A1 |
| | | 4 |

## 15. 9709/32/F/M/20 Q2

| Question | Answer | Marks |
|---|---|---|
| 2 | Use law of logarithm of a power and sum and remove logarithms | M1 |
| | Obtain a correct equation in any form, e.g. $3(2x+5) = (x+2)^2$ | A1 |
| | Use correct method to solve a 3-term quadratic, obtaining at least one root | M1 |
| | Obtain final answer $x = 1 + 2\sqrt{3}$ or $1 + \sqrt{12}$ only | A1 |
| | | 4 |

### 16. 9709/31/M/J/20 Q1

| Question | Answer | Marks |
|---|---|---|
| 1 | Use law of the logarithm of a product or power | M1 |
| | Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1 - 2x)\ln 3 < x \ln 5$ | A1 |
| | Solve for $x$ | M1 |
| | Obtain $x > \dfrac{\ln 6}{\ln 45}$ | A1 |
| | | 4 |

### 17. 9709/32/M/J/20 Q2

| Question | Answer | Marks |
|---|---|---|
| 2 | State or imply $2\ln y = \ln A + kx$ | B1 |
| | Substitute values of $\ln y$ and $x$, or equate gradient of line to $k$, and solve for $k$ | M1 |
| | Obtain $k = 0.80$ | A1 |
| | Solve for $\ln A$ | M1 |
| | Obtain $A = 3.31$ | A1 |
| | **Alternative method for question 2** | |
| | Obtain two correct equations in $y$ and $x$ by substituting $y$- and $x$- values in the given equation | B1 |
| | Solve for $k$ | M1 |
| | Obtain $k = 0.80$ | A1 |
| | Solve for $A$ | M1 |
| | Obtain $A = 3.31$ | A1 |
| | | 5 |

### 18. 9709/33/M/J/20 Q3

| Question | Answer | Marks |
|---|---|---|
| 3(a) | Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent | B1 |
| | Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent | B1 |
| | | 2 |
| 3(b) | Solve a 3-term quadratic for $u$ | M1 |
| | Obtain root $\dfrac{1}{2}\left(-1 + \sqrt{5}\right)$, or decimal in [0.61, 0.62] | A1 |
| | Use correct method for finding $x$ from a positive root | M1 |
| | Obtain answer $x = -0.481$ only | A1 |
| | | 4 |

## 19. 9709/31/O/N/20 Q4

| 4 | State or imply $\log_{10} 10 = 1$ | B1 | $\log_{10} 10^{-1} = -1$ |
|---|---|---|---|
| | Use law of the logarithm of a power, product or quotient | M1 | |
| | Obtain a correct equation in any form, free of logs | A1 | e.g. $(2x+1)/(x+1)^2 = 10^{-1}$ or $10(2x+1)/(x+1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$ |
| | Reduce to $x^2 - 18x - 9 = 0$, or equivalent | A1 | |
| | Solve a 3-term quadratic | M1 | |
| | Obtain final answers $x = 18.487$ and $x = -0.487$ | A1 | Must be 3 d.p. Do not allow rejection. |
| | | 6 | |

## 20. 9709/32/O/N/20 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | State that $1 + e^{-3x} = e^2$ | B1 | With no errors seen to that point |
| | Use correct method to solve an equation of the form $e^{-3x} = a$, where $a > 0$, for $x$ or equivalent | M1 | $\left(e^{-3x} = 6.389\ldots\right)$ Evidence of method must be seen. |
| | Obtain answer $x = -0.618$ only | A1 | Must be 3 decimal places |
| | **Alternative method for question 1** | | |
| | State that $1 + e^{-3x} = e^2$ | B1 | |
| | Rearrange to obtain an expression for $e^x$ and solve an equation of the form $e^x = a$, where $a > 0$, or equivalent | M1 | $e^x = \sqrt[3]{\dfrac{1}{e^2 - 1}}$ |
| | Obtain answer $x = -0.618$ only | A1 | Must be 3 decimal places |
| | | 3 | |

## 21. 9709/33/O/N/20 Q4

| 4 | State or imply $\log_{10} 10 = 1$ | B1 | $\log_{10} 10^{-1} = -1$ |
|---|---|---|---|
| | Use law of the logarithm of a power, product or quotient | M1 | |
| | Obtain a correct equation in any form, free of logs | A1 | e.g. $(2x+1)/(x+1)^2 = 10^{-1}$ or $10(2x+1)/(x+1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$ |
| | Reduce to $x^2 - 18x - 9 = 0$, or equivalent | A1 | |
| | Solve a 3-term quadratic | M1 | |
| | Obtain final answers $x = 18.487$ and $x = -0.487$ | A1 | Must be 3 d.p. Do not allow rejection. |
| | | 6 | |

## 22. 9709/32/F/M/21 Q1

| Question | Answer | Marks |
|---|---|---|
| 1 | Use law of the logarithm of a product or power | M1 |
| | Obtain a correct equation free of logarithms, e.g. $3(x^3 - 3) = x^3$ | A1 |
| | Obtain $x = 1.65$ | A1 |
| | | 3 |

## 23. 9709/31/M/J/21 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Reduce to a 3-term quadratic $u^2 + 6u - 1 = 0$ OE | B1 | Allow '= 0' implied |
| | Solve a 3-term quadratic for $u$ | M1 | |
| | Obtain root $\sqrt{10} - 3$ | A1 | |
| | Obtain answer $x = -1.818$ only | A1 | The question asks for 3 d.p. |
| | Reject $-\sqrt{10} - 3$ correctly | B1 | e.g. by stating that $e^x > 0$ or $\ln(-10 - \sqrt{3})$ is impossible Not "math error". |
| **Alternative method for Question 2** | | | |
| | Rearrange to obtain a correct iterative formula | B1 | e.g. $x_{n+1} = -\ln(6 + e^{x_n})$ |
| | Use the iterative process at least twice | M1 | |
| | Obtain answer $x = -1.818$ | A1 | |
| | Show sufficient iterations to at least 4 d.p. to justify $x = -1.818$ | A1 | $1, -2.165..., -1.811..., -1.819..., -1.818..., -1.818...$ |
| | Clear explanation of why there is only one real root | B1 | |
| | | 5 | |

## 24. 9709/32/M/J/21 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | State or imply $\ln x = \ln A - y \ln 3$ | B1 | $\left( y = -\dfrac{1}{\ln 3} \ln x + \dfrac{\ln A}{\ln 3} \right)$ |
| | State that the graph of $y$ against $\ln x$ has an equation that is *linear* in $y$ and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line | B1 | Must be a correct statement. Accept if the 2 equations are written side by side with no comment. An equation with $y \ln 3$ should be compared with the form $py + q \ln x = c$. |
| | State that the gradient is $-\dfrac{1}{\ln 3}$ | B1 | OE. Exact answer required. ISW after a correct statement. |
| | | 3 | |
| 3(b) | Substitute $\ln x = 0$, $y = 1.3$ and use correct method to solve for $A$ | M1 | $(\ln A = 1.3 \ln 3)$ Follow *their* equation in $y$ and $\ln x$. Must be substituting $\ln x = 0$, not $x = 0$. $\ln 0$ 'used' in the solution scores M0A0. |
| | Obtain answer $A = 4.17$ only | A1 | Must be 2 d.p. as specified in question |
| | | 2 | |

**25. 9709/33/M/J/21 Q2**

| Question | Answer | Marks |
|---|---|---|
| 2 | State or imply $u^2 - 3u - 1 = 0$, or equivalent in $4^x$ | B1 |
| | Solve for $u$ or $4^x$ | M1 |
| | Obtain root $\frac{1}{2}\left(3 + \sqrt{13}\right)$, or decimal in [3.30, 3.31] | A1 |
| | Use correct method for finding $x$ from a positive root | M1 |
| | Obtain answer $x = 0.862$ and no other | A1 |
| | | 5 |

**26. 9709/31/O/N/21 Q1**

| Question | Answer | Marks |
|---|---|---|
| 1 | State or imply non-modular equation $4^2\left(5^x - 1\right)^2 = \left(5^x\right)^2$ or pair of equations $4\left(5^x - 1\right) = \pm 5^x$ | M1 |
| | Obtain $5^x = \frac{4}{3}$ and $5^x = \frac{4}{5}$ (or $5^{x+1} = 4$) | A1 |
| | Use correct method for solving an equation of the form $5^x = a$, or $5^{x+1} = b$ where $a > 0$, or $b > 0$ | M1 |
| | Obtain answers $x = 0.179$ and $x = -0.139$ | A1 |
| | **Alternative method for question 1** | |
| | Obtain $5^x = \frac{4}{3}$ by solving an equation | B1 |
| | Obtain $5^x = \frac{4}{5}$ (or $5^{x+1} = 4$) by solving an equation | B1 |
| | Use correct method for solving an equation of the form $5^x = a$, or $5^{x+1} = b$ where $a > 0$, or $b > 0$ | M1 |
| | Obtain answers $x = 0.179$ and $x = -0.139$ | A1 |
| | | 4 |

### 27. 9709/32/O/N/21 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation | B1 | e.g. $5x^2 + 22x + 8 = 0$ <br> Allow recovery from 'invisible brackets' on RHS |
| | Form and solve a 3-term quadratic in $x$ | M1 | |
| | Obtain critical values $x = -4$ and $x = -\dfrac{2}{5}$ | A1 | |
| | State final answer $x < -4$, $x > -\dfrac{2}{5}$ | A1 | Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. <br> Allow 'or' but not 'and'. <br> $-\dfrac{2}{5} < x < -4$ scores A0. <br> Accept equivalent forms using brackets e.g. <br> $x \in (-\infty, -4) \cup (-0.4, \infty)$ |
| **Alternative method for Question 1** | | | |
| | Obtain critical value $x = -4$ from a graphical method, or by solving a linear equation or linear inequality | B1 | |
| | Obtain critical value $x = -\dfrac{2}{5}$ similarly | B2 | |
| | State final answer $x < -4$, $x > -\dfrac{2}{5}$ | B1 | Do not condone $\leqslant$ for $<$, or $\geqslant$ for $>$ in the final answer. <br> Allow 'or' but not 'and'. <br> $-\dfrac{2}{5} < x < -4$ scores A0. <br> Accept equivalent forms using brackets e.g. <br> $x \in (-\infty, -4) \cup (-0.4, \infty)$ |
| | | 4 | |

### 28. 9709/33/O/N/21 Q2

| Question | Answer | Marks |
|---|---|---|
| 2 | State or imply $u^2 - 3u - 1 = 0$, or equivalent in $4^x$ | B1 |
| | Solve for $u$ or $4^x$ | M1 |
| | Obtain root $\dfrac{1}{2}\left(3 + \sqrt{13}\right)$, or decimal in [3.30, 3.31] | A1 |
| | Use correct method for finding $x$ from a positive root | M1 |
| | Obtain answer $x = 0.862$ and no other | A1 |
| | | 5 |

For more topical past papers and revision notes visit *exambuddy.org*

### 29. 9709/32/F/M/22 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | State or imply $n \ln x + 2 \ln y = \ln C$ | B1 | |
| | Substitute values of $\ln y$ and $\ln x$, or equate gradient of line to $\pm\frac{1}{2}n$, but not $\pm n$, and solve for $n$ | M1 | Using ln$x$ and ln$y$ values |
| | Obtain $n = 0.8[0]$ or $0.8[00]$ or $\frac{4}{5}$ | A1 | |
| | Solve for $C$ | M1 | Using ln$x$ and ln$y$ values in equation of correct form, that is $\ln C$ not $C$. Allow $C = e^{2.668}$. |
| | Obtain $C = 14.41$ | A1 | Must be 2 d.p. |
| | **Alternative method for question 3** | | |
| | Obtain two correct equations in $n$ and $C$ by substituting $x$ and $y$ values in the given equation | B1 | $(2.886)^n \times (2.484)^2 = C$ and $(1.363)^n \times (3.353)^2 = C$ |
| | Solve for $n$ | M1 | Using $x$ and $y$ values |
| | Obtain $n = 0.8[0]$ or $0.8[00]$ or 4/5 | A1 | $\left(\frac{2.886}{1.363}\right)^n \times \left(\frac{2.484}{3.353}\right)^2 = 1$ leading to $n = 0.7995$ |
| | Solve for $C$ | M1 | Using $x$ and $y$ values |
| | Obtain $C = 14.41$ | A1 | Must be 2 d.p. |
| | | 5 | |

### 30. 9709/31/M/J/22 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use law of the logarithm of a product or a quotient or a power | *M1 | |
| | Obtain a correct linear equation in any form | A1 | e.g. $\ln 2 + (2x-1)\ln 3 = (x+1)\ln 4$ or $\log_2 2 + (2x-1)\log_2 3 = (2x+2)\log_2 2$ |
| | Solve for $x$ | DM1 | Allow for unsimplified expression $x = ...$ Allow M1 M1 for $x=1.45$ from $6^{2x-1} = 4^{x+1}$. |
| | Obtain answer $x = 2.21$ | A1 | The question asks for 2 dp. |
| | **Alternative method for question 1** | | |
| | Correct use of indices to obtain $2.25^x = 6$ or $1.5^{2x} = 6$ | M1 A1 | |
| | Correct use of logarithms to solve for $x$ | M1 | Allow solution of $2.25^x = 6$ by trial and improvement as far as 2.2... |
| | Obtain answer $x = 2.21$ | A1 | Need to see an intermediate step / sequence of iterations. |
| | | 4 | |

### 31. 9709/32/M/J/22 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use law of the logarithm of a product, power or quotient or a law of indices (on an expression that is relevant to the question) | M1 | e.g. $\ln(e^{2x}+3)-\ln 3=\ln\left(\dfrac{e^{2x}+3}{3}\right)$ or $e^{(2x+\ln 3)}=e^{2x}e^{\ln 3}$ |
| | State a correct equation without logs (in any form) | A1 | e.g. $3+e^{2x}=3e^{2x}$ |
| | Carry out correct method to solve an equation of the form $e^{2x}=a$, where $a>0$, or for solving $e^x=b$ $(b>0)$ if they have already taken the square root | M1 | Allow for $x=\frac{1}{2}\ln\frac{3}{2}$. M1 can be implied by correct answer. |
| | Obtain answer $x=0.203$ | A1 | CAO. The question requires 3 d.p. Answer only with no working shown is 0/4. |
| | | 4 | |

### 32. 9709/33/M/J/22 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Use law of logarithm of a product or power | M1 | One correct application of a log law. |
| | Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2=\ln 5-x\ln 3$ | A1 | |
| | Solve for $x$ | M1 | As far as $x=\ldots$ with only minor slips in processing. |
| | Obtain answer $x=\dfrac{\ln 10}{\ln 24}$ | A1 | |
| | **Alternative method for question 3** | | |
| | Use laws of indices to split at least one exponential term | M1 | e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with $8^x$ |
| | Obtain $24^x=10$ | A1 | OE |
| | Solve for $x$ | M1 | |
| | Obtain answer $x=\dfrac{\ln 10}{\ln 24}$ | A1 | |
| | | 4 | |

### 33. 9709/31/O/N/22 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Use law of logarithm of a power | M1 | $\log_3(2x+1)=1+\log_3(x-1)^2$ |
| | Use $\log_3 3=1$ | B1 | $\log_3(2x+1)=\log_3 3+2\log_3(x-1)$ $\left[\log 3\left(\dfrac{2x+1}{(x-1)^2}\right)=\log_3 3 \quad \text{or} \quad \left(\dfrac{2x+1}{(x-1)^2}\right)=3\right]$ **SC** For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2, and confusing coefficients, allow $\log_3(\ldots)=c$ leading to $(\ldots)=3^c$ **B1**. |
| | Obtain $3x^2-8x+2=0$ or $1.5x^2-4x+1=0$ | A1 | OE 3 terms only and $=0$ required. |
| | | 3 | |
| 3(b) | Solve 3-term quadratic equation from part **3(a)** or restart to find $y$ | M1 | $y=\dfrac{4\pm\sqrt{10}}{6}$ or $y=1.1937\ldots$ or $y=0.1396\ldots$ $(x=2.3874$ or $x=0.2792)$ May solve for $x$ but must find $y=\dfrac{x}{2}$ to gain M1. |
| | Obtain answer 1.19 | A1 | CAO. 2 dp required. |
| | | 2 | |

## 34. 9709/32/O/N/22 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use law of the logarithm of a power or product | M1 | Ignoring the 3 or the 5 is not a misread. |
| | Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 + (1-x)\ln 3$ | A1 | Condone invisible brackets if they are used correctly later. |
| | Solve for $x$ | M1 | Get as far as $x = \ldots$ Condone minor slips in the processing e.g. sign errors and losing a term that had been there, but award M0 for a fundamental error e.g. $3x\ln 2 + x\ln 3 = 3x\ln 6$ or ignoring the 3 or the 5 completely. Condone working in decimals. |
| | Obtain **final** answer $x = \dfrac{\ln 30}{\ln 24}$ | A1 | Do not ISW |
| | **Alternative method for question 1** | | |
| | Use laws of indices to split at least one exponential term | M1 | e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with $8^x$ and/or $3^x$. |
| | Obtain $24^x = 30$ | A1 | Or equivalent e.g. $3^x 8^x = 30$ not for $3^x 2^{3x} = 30$ (need two factors with the same index). |
| | Solve for $x$ | M1 | Get as far as $x = \ldots$ |
| | Obtain **final** answer $x = \dfrac{\ln 30}{\ln 24}$ | A1 | Do not ISW |
| | | 4 | |

## 35. 9709/33/O/N/22 Q1

| Question | Answer | Marks |
|---|---|---|
| 1 | Use law for the logarithm of a product, quotient or power | M1 |
| | Remove logarithms and state a correct equation, e.g. $x(2x-1) = (x+1)^2$ | A1 |
| | Solve a 3-term quadratic obtaining at least one root | M1 |
| | Obtain answer 3.303 only | A1 |
| | | 4 |

## 36. 9709/32/F/M/23 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use law of the logarithm of a quotient or express $x$ as $\ln e^x$ | M1 | $x = \ln[(2y-3)/(y+4)]$ or $\ln e^x = \ln(2y-3) - \ln(y+4)]$. |
| | Remove logarithms and obtain a correct equation e.g. $e^x = \dfrac{2y-3}{y+4}$ | A1 | |
| | Obtain answer $y = \dfrac{3+4e^x}{2-e^x}$ | A1 | OE ISW |
| | | 3 | |

## 37. 9709/31/M/J/23 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $3\left(e^{2x}\right)^2 - 5\left(e^{2x}\right) - 4 = 0$ | B1 | OE Form 3 term quadratic in $e^{2x}$. |
| | $e^{2x} = \dfrac{5 \pm \sqrt{73}}{6}, \quad x = \dfrac{1}{2}\ln\left(\dfrac{5 + \sqrt{73}}{6}\right)$ | M1 | Use correct method to solve for $x$. |
| | $x = 0.407$ | A1 | Only |
| | | 3 | |

## 38. 9709/32/M/J/23 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use law of the logarithm of a power, quotient or product | M1 | Must be used correctly on a **correct** term. e.g. M1 for $2\ln x = \ln x^2$ but M0 for $2\ln x - \ln 2 = 2\ln \frac{x}{2}$. M0 for $\ln(2x^2 - 3) = \ln 2x^2 - \ln 3 = \ln 2 + 2\ln x - \ln 3$. |
| | Remove logarithms and obtain a correct equation in $x$ | A1 | e.g. $2x^2 - 3 = \dfrac{x^2}{2}$. |
| | Obtain final answer $x = \sqrt{2}$ only | A1 | If $x = -\sqrt{2}$ is mentioned, it must be rejected. |
| | | 3 | |

## 39. 9709/33/M/J/23 Q1

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Use exponentials or law for the logarithm of a product, quotient or power | M1* | $e^{\ln(5+x)} = e^{5+\ln x}$ insufficient. Need e.g. $\ln\left(\dfrac{x+5}{x}\right) = 5$ or $\ln(x+5) = \ln\left(e^5\right) + \ln x$ or $\ln(x+5) = \ln(e^5 x)$ or $x + 5 = e^{5+\ln x}$ or $x + 5 = e^5 e^{\ln x}$ and others. |
| | Correctly remove logarithms | DM1 | |
| | Obtain a correct equation in $x$ | A1 | e.g. $\dfrac{x+5}{x} = e^5$ (or $148.4\ldots$) or $x + 5 = xe^5$. |
| | Obtain $0.034$ | A1 | CAO Final answer must be 3d.p. |
| | | 4 | |

### 40. 9709/31/O/N/23 Q3

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | State or imply that $\ln y = \ln a + x \ln b$ | B1 | |
| | Carry out a completely correct method for finding $\ln a$ or $\ln b$ | M1 | $3.7 = \ln a + \ln b$ and $6.46 = \ln a + 2.2 \ln b$ leading to $\ln a = 1.4$, $\ln b = 2.3$. |
| | Obtain value $a = 4.06$ | A1 | |
| | Obtain value $b = 9.97$ | A1 | SC B1 for $a = e^{1.4}$ and $b = e^{2.3}$. |
| | **Alternative Method for Question 3** | | |
| | $e^{3.7} = ab^1$ and $e^{6.46} = ab^{2.2}$ | B1 | |
| | Divide to obtain $e^{2.76} = b^{1.2}$ and state or imply $2.76 = 1.2 \ln b$ | M1 | |
| | Obtain value $a = 4.06$ | A1 | |
| | Obtain value $b = 9.97$ | A1 | |
| | | 4 | |

### 41. 9709/32/O/N/23 Q2

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Obtain $\dfrac{dx}{dt} = \dfrac{2}{t} \ln t$ | B1 | Any equivalent form. |
| | Obtain $\dfrac{dy}{dt} = -2te^{2-t^2}$ | B1 | Any equivalent form. |
| | $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$ and substitute $t = e$ | M1 | Correct use of chain rule for $\dfrac{dy}{dx}\left(\dfrac{-2e^2 e^{2-e^2}}{2\ln e}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e. |
| | Obtain $-e^{4-e^2}$ | A1 | ISW Accept $-0.0337(405..)$. Accept $-e^4 e^{-e^2}$, $\dfrac{-e^4}{e^{e^2}}$ and $-e^2 e^{2-e^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen. |
| | | 4 | |

### 42. 9709/33/O/N/23 Q1

PUBLISHED

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | State or imply non-modular inequality $-0.5 < 2^{x+1} - 2 < 0.5$, can be in two separate statements, or $\left(2^{x+1} - 2\right)^2 < 0.5^2$ or corresponding pair of linear equations $0.5 = 2^{x+1} - 2$ and $-0.5 = 2^{x+1} - 2$ or quadratic equation $\left(2^{x+1} - 2\right)^2 = 0.5^2$ | B1 | $-0.25 < 2^x - 1 < 0.25$, can be in two separate statements, or $\left(2^x - 1\right)^2 < 0.25^2$ or corresponding pair of linear equations $0.25 = 2^x - 1$ and $-0.25 = 2^x - 1$ or quadratic equation $\left(2^x - 1\right)^2 = 0.25^2$. Incorrect inequality mark recoverable by correct final answer or $x < 0.32$ and $x > -0.42$. |
| | Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$ | M1 | Reach $(x + 1)\ln 2 = \ln a$ or equivalent, do not need to reach $x = \ldots$ |
| | Obtain critical values $x = 0.322$ and $-0.415$ or awrt $x = 0.32$ and $-0.42$ or exact equivalents | A1 | e.g. $\dfrac{\ln 2.5}{\ln 2} - 1$ and $\dfrac{\ln 1.5}{\ln 2} - 1$. |
| | State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$ | A1 | Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4. |
| | **Alternative method for Question 1** | | |
| | Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$ | M1 | May see $2^{x+1} = 1.5$ and $2^{x+1} = 2.5$. Reach $(x + 1)\ln 2 = \ln a$ or equivalent, don't need to reach $x = \ldots$ |
| | Obtain one critical value, e.g. 0.322 or awrt $x = 0.32$ or exact equivalent | A1 | e.g. $\dfrac{\ln 2.5}{\ln 2} - 1$. |
| | Obtain the other critical value e.g. $-0.415$ or awrt $x = -0.42$ or exact equivalent | A1 | e.g. $\dfrac{\ln 1.5}{\ln 2} - 1$. |

### 43. 9709/32/F/M/24 Q4

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Obtain $\ln p - \ln q = a$ | B1 | $\dfrac{p}{q} = e^a$. |
| | Obtain $\ln p + 2\ln q = b$ | B1 | $pq^2 = e^b$. |
| | Completed method to obtain $\ln\left(p^7 q\right)$ | M1 | E.g. $\ln q = \dfrac{b - a}{3}$, $\ln p = \dfrac{2a + b}{3}$ and attempt $7\ln p + \ln q$. All exponentials must be removed to obtain **M1**. |
| | Obtain $\dfrac{13a + 8b}{3}$ | A1 | |
| | **Alternative solution for Question 4** | | |
| | State $p^7 q = \left(\dfrac{p}{q}\right)^x \left(q^2 p\right)^y$ | B1 | Or $\ln p^7 q = x\ln\dfrac{p}{q} + y\ln q^2 p$. |
| | Equate indices to form simultaneous equations in $x$ and $y$, can have errors | M1 | $x + y = 7$ and $-x + 2y = 1$. |
| | Obtain $7 = x + y$ and $1 = 2y - x$ | A1 | Leading to $x = \frac{13}{3}$, $y = \frac{8}{3}$. |
| | Evaluate $x \times a + y \times b$ to obtain $\dfrac{13a + 8b}{3}$ | A1 | |
| | | 4 | |