1. 9709/32/F/M/18 Q4

The variables x and y satisfy the equation $y^n = Ax^3$, where n and A are constants. It is given that $y = 2.58$ when $x = 1.20$, and $y = 9.49$ when $x = 2.51$.

(i) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [2]

(ii) Find the values of n and A, giving your answers correct to 2 decimal places. [4]

2. 9709/31/M/J/18 Q1

Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4 \ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

3. 9709/32/M/J/18 Q1

Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

4. 9709/33/M/J/18 Q1

Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

5. 9709/31/O/N/18 Q2

Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

6. 9709/32/O/N/18 Q4

Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

7. 9709/33/O/N/18 Q2

Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

8. 9709/32/F/M/19 Q1

(i) Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

9. 9709/31/M/J/19 Q2

Showing all necessary working, solve the equation $\ln(2x - 3) = 2\ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

10. 9709/32/M/J/19 Q2

Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

11. 9709/33/M/J/19 Q1

Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

12. 9709/31/O/N/19 Q1

Given that $\ln\left(1+e^{2y}\right)=x$, express y in terms of x. [3]

13. 9709/32/O/N/19 Q1

Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

14. 9709/33/O/N/19 Q3

Showing all necessary working, solve the equation $\frac{3^{2x}+3^{-x}}{3^{2x}-3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]

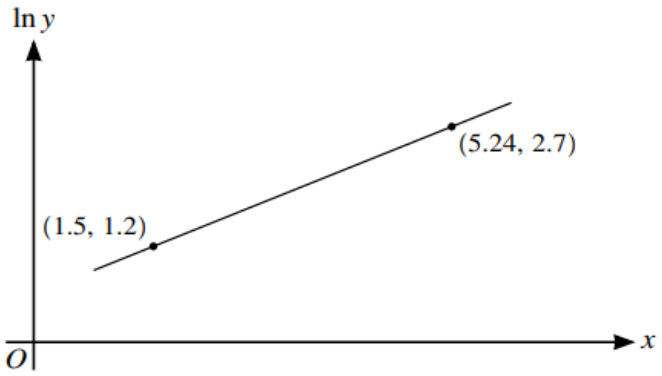
15. 9709/32/F/M/20 Q2

Solve the equation $\ln 3 + \ln(2x + 5) = 2\ln(x + 2)$. Give your answer in a simplified exact form. [4]

16. 9709/31/M/J/20 Q1

Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

17. 9709/32/M/J/20 Q2



The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

18. 9709/33/M/J/20 Q3

(a) Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

19. 9709/31/O/N/20 Q4

Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

20. 9709/32/O/N/20 Q1

Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places. [3]

21. 9709/33/O/N/20 Q4

Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

22. 9709/32/F/M/21 Q1

Solve the equation $\ln(x^3 - 3) = 3\ln x - \ln 3$. Give your answer correct to 3 significant figures. [3]

23. 9709/31/M/J/21 Q2

Find the real root of the equation $\frac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

24. 9709/32/M/J/21 Q3

The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

25. 9709/33/M/J/21 Q2

Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

26. 9709/31/O/N/21 Q1

Solve the equation $4|5^x - 1| = 5^x$, giving your answers correct to 3 decimal places. [4]

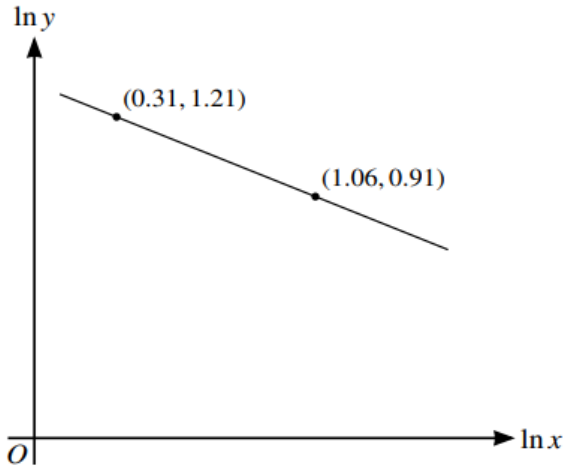
27. 9709/32/O/N/21 Q1

Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

28. 9709/33/O/N/21 Q2

Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

29. 9709/32/F/M/22 Q3



The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0.31, 1.21) and (1.06, 0.91), as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

30. 9709/31/M/J/22 Q1

Solve the equation $2(3^{2x-1}) = 4^{x+1}$, giving your answer correct to 2 decimal places. [4]

31. 9709/32/M/J/22 Q1

Solve the equation $\ln(e^{2x} + 3) = 2x + \ln 3$, giving your answer correct to 3 decimal places. [4]

32. 9709/33/M/J/22 Q3

(a) Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x. [3]

(b) Hence solve the equation $\log_3(4y + 1) = 1 + 2\log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

33. 9709/31/O/N/22 Q3

Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers.

[4]

34. 9709/32/O/N/22 Q1

Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

35. 9709/33/O/N/22 Q1

Solve the equation $\ln(2x-1) = 2\ln(x+1) - \ln x$. Give your answer correct to 3 decimal places. [4]

36. 9709/32/F/M/23 Q1

It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

37. 9709/31/M/J/23 Q1

Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

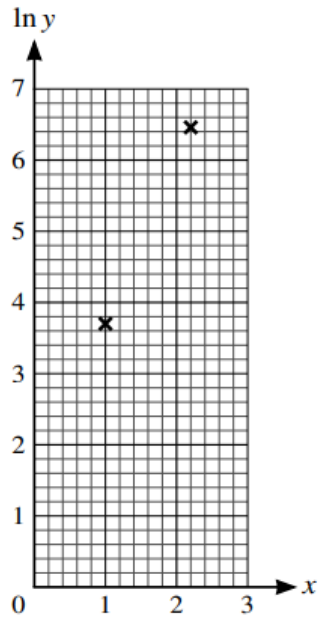
38. 9709/32/M/J/23 Q2

Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

39. 9709/33/M/J/23 Q1

Solve the equation $\ln(x+5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

40. 9709/31/O/N/23 Q3



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

41. 9709/32/O/N/23 Q2

The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = c$, simplifying your answer. [4]

42. 9709/33/O/N/23 Q1

Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]

43. 9709/32/F/M/24 Q4

The positive numbers p and q are such that

$$\ln\left(\frac{p}{q}\right) = a \quad \text{and} \quad \ln\left(q^2 p\right) = b.$$

Express $\ln\left(p^7 q\right)$ in terms of a and b. [4]