1.

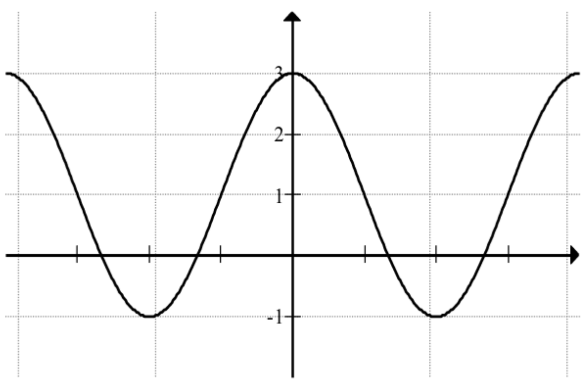
9 (i)	$2\cos x\cot x = \cot x + 2\cos x$		
	$2\cos x\dfrac{\cos x}{\sin x} + 1 = \dfrac{\cos x}{\sin x} + 2\cos x$	**M1**	for use of $\cot x = \dfrac{\cos x}{\sin x}$ for both terms
	$2\cos^2 x + \sin x = \cos x + 2\cos x\sin x$	**DM1**	for multiplication throughout by $\sin x$
	$2\cos^2 x - 2\cos x\sin x = \cos x - \sin x$		
	$2\cos x(\cos x - \sin x) = \cos x - \sin x$	**DM1**	for attempt to factorise
	$(2\cos x - 1)(\cos x - \sin x) = 0$	**A1**	for completely correct solution www
	Alternative method:		
	$a\cos^2 x - a\cos x\sin x - b\cos x + b\sin x = 0$	**M1**	for expansion of RHS
	$a\cos x\cot x - a\cos x - b\cot x + b = 0$	**DM1** **DM1**	for division by $\sin x$ for comparing like terms to obtain both *a* and *b*
	$a = 2,\quad b = 1$	**A1**	for both correct www
(ii)	$(2\cos x - 1)(\cos x - \sin x) = 0$		
	$\cos x = \dfrac{1}{2}$, $\tan x = 1$	**M1**	for either
	$x = \dfrac{\pi}{3}$, $x = \dfrac{\pi}{4}$	**A1,A1**	**A1** for each, penalise extra solutions within the range by withholding the last A mark
	Alternative method: $(2\cos x - 1)(\cot x - 1) = 0$		
	Leading to $\cos x = \dfrac{1}{2}$, $\tan x = 1$	**M1**	for attempt to factorise the original equation and attempt to solve
	$x = \dfrac{\pi}{3}$, $x = \dfrac{\pi}{4}$	**A1,A1**	**A1** for each, penalise extra solutions within the range by withholding the last A mark

2.

5 (i)	$(1-\cos\theta)(1+\sec\theta)$		
	$= 1 - \cos\theta + \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta}$	**M1**	M1 for expansion and use of $\sec\theta = \dfrac{1}{\cos\theta}$ consistently, allow one sign error
	$= \sec\theta - \cos\theta$		
	$= \dfrac{1}{\cos\theta} - \cos\theta$		
	$= \dfrac{1-\cos^2\theta}{\cos\theta}$	**DM1**	for attempt at a single fraction, dependent on first **M1**
	$= \dfrac{\sin^2\theta}{\cos\theta}$	**A1**	
	$= \sin\theta\tan\theta$ www	**A1**	

	Alternative method:		
	$(1-\cos\theta)\left(\dfrac{\cos\theta+1}{\cos\theta}\right)$	**M1**	for attempt at a single fraction for second factor and use of $\sec\theta = \dfrac{1}{\cos\theta}$
	$= \dfrac{1-\cos^2\theta}{\cos\theta}$	**DM1**	for expansion
	$= \dfrac{\sin^2\theta}{\cos\theta}$	**A1**	
	$= \sin\theta\tan\theta$ www	**A1**	
(ii)	$\sin\theta\tan\theta = \sin\theta$ $\sin\theta(\tan\theta - 1) = 0$		
	$\tan\theta = 1,\ \theta = \dfrac{\pi}{4}$, allow 0.785 or better	**B1**	for $\theta = \dfrac{\pi}{4}$ from $\tan\theta = 1$
	$\sin\theta = 0,\ \theta = 0,\ \pi$ or 3.14 or better	**B1** **B1**	for $\theta = 0$ from $\sin\theta = 0$ for $\theta = \pi$ from $\sin\theta = 0$

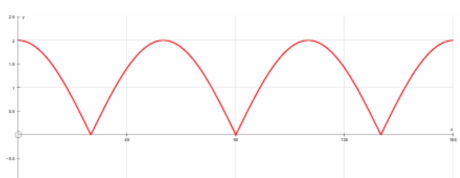
3.

9 (a)	$a = 2 \quad b = 4 \quad c = -2$	**B3**	**B1** for each correct value
(b) (i)		**B3,2,1, 0**	sinusoidal curve symmetrical about y-axis clear intent to have amplitude of 2 2 cycles If not fully correct max **B2**
(ii)	$-\dfrac{\pi}{2}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{\pi}{2}, -\dfrac{\pi}{3}, \dfrac{\pi}{3}$ cao	**B2**	**B1** for any 4 correct

4.

12 (i)	$8(1-\cos^2 A)+2\cos A=7$ or better	**B1**	
	Solves or factorises *their* 3-term quadratic in cos$A$	**M1**	
	60, 104.477... rounded or truncated to 1 dp or more;	**A2**	with no extras in range; not from clearly wrong working but allow recovery from minor slips or **A1** for either, ignoring extras
(ii)	$\sin(3B+1)=0.4$ soi	**B1**	may be implied by $\frac{1}{\sin(3B+1)}=2.5$
	[3B + 1 =] 0.41 or better	**M1**	implies **B1**
	0.577, 1.9[0], 2.67 or 0.57669..., 1.89823... , 2.67108... rounded or truncated to 4 or more sf	**A2**	with no extras in range; or **A1** for any one correct ignoring extras If **M0** then **B2** for all 3 correct angles found or **B1** for 1 or 2 correct angles found

5.

1		**B1**	for symmetrical shape as in the diagram with curved maxima of equal height and cusps on the x-axis
		B1	for a complete 'curve' with all low points on the x-axis and all high points on $y=2$
		B1	for a complete 'curve' meeting the x-axis at $x=30°$, $90°$, $150°$ only.

6.

8 (a) (i)	$\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta - \sin\theta} = \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}$	**M1**	for using $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and either attempt to multiply top and bottom by sin θ or an attempt to combine terms in denominator.
	$= \dfrac{1}{1-\sin^2\theta}$ or $= \dfrac{\frac{1}{\sin\theta}}{\frac{(1-\sin^2\theta)}{\sin\theta}}$	**DM1**	for correct use of $1\text{-}\sin^2\theta = \cos^2\theta$
	$= \dfrac{1}{\cos^2\theta}$ $= \sec^2\theta$	**A1**	for completing the proof
	Alternative Method using cosec $\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta - \sin\theta} = \dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta - \frac{1}{\text{cosec}\,\theta}}$		
	$= \dfrac{\text{cosec}^2\theta}{\text{cosec}^2\theta - 1}$	**M1**	for using $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ and an attempt to combine terms in denominator.
	$= \dfrac{1+\cot^2\theta}{\cot^2\theta}$	**DM1**	for use of $1+\cot^2\theta = \text{cosec}^2\theta$
	$= \tan^2\theta + 1 = \sec^2\theta$	**A1**	for completing the proof
(ii)	$\cos^2\theta = \dfrac{1}{4}, \quad \cos\theta = \pm\dfrac{1}{2}$ or $\tan^2\theta = 3, \quad \tan\theta = \pm\sqrt{3}$ or $\sin^2\theta = \dfrac{3}{4}, \quad \sin\theta = \pm\dfrac{\sqrt{3}}{2}$	**M1**	for using (i) to obtain a value for $\cos^2\theta$, $\tan^2\theta$ or $\sin^2\theta$ and then taking the square root.
	$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$	**A1** **A1**	for two correct values for two further correct values and no extras in range.
(b)	$\tan\left(x + \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}}$ $x = \dfrac{\pi}{6} - \dfrac{\pi}{4}, \dfrac{7\pi}{6} - \dfrac{\pi}{4}, \dfrac{13\pi}{6} - \dfrac{\pi}{4}$	**M1**	for correct order of operations, can be implied by $x = -\dfrac{\pi}{12}$
	$x = \left(-\dfrac{\pi}{12}\right), \dfrac{11\pi}{12}, \dfrac{23\pi}{12}$	**A1,A1**	**A1** for $x = \dfrac{11\pi}{12}$ **A1** for $x = \dfrac{23\pi}{12}$ If there are extra solutions in range in addition to the two correct ones then **A1A0**

7.

6 **(i)**	$\frac{\cos x}{1+\tan x}-\frac{\sin x}{1+\cot x}=\frac{\cos x}{1+\frac{\sin x}{\cos x}}-\frac{\sin x}{1+\frac{\cos x}{\sin x}}$	**M1**	$\tan x=\frac{\sin x}{\cos x}$ and $\cot x=\frac{\cos x}{\sin x}$
	$=\frac{\cos^2 x}{\cos x+\sin x}-\frac{\sin^2 x}{\cos x+\sin x}$	**M1** **A1**	Attempt to multiply by $\cos x$ and $\sin x$
	$=\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\cos x+\sin x)}$	**A1**	AG
(ii)	$-\sin x+\cos x=3\sin x-4\cos x$	**M1**	equate and collect $\sin x$ and $\cos x$ oe
	$5\cos x=4\sin x$		
	$\tan x=\frac{5}{4}$	**A1**	
	$x=51.3°, -128.7°$	**A1A1**	**FT** from $\tan x=k$

8.

Question	Answer	Marks	Partial Marks
6(i)	$\frac{1}{\sin\theta}\times\frac{1}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$	**M1**	for $\cot\theta=\frac{\cos\theta}{\sin\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
	dealing with the fractions correctly	**M1**	
	$\frac{1}{\sin\theta}\times\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$	**M1**	use of identity
	$=\cos\theta$	**A1**	correct simplification, with all correct
	Alternative 1 $\frac{\operatorname{cosec}\theta}{\frac{1}{\tan\theta}(1+\tan^2\theta)}$	**M1**	dealing with fractions
	$=\frac{\tan\theta\operatorname{cosec}\theta}{\sec^2\theta}$	**M1**	use of appropriate identity
	$=\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin\theta}\times\cos^2\theta$	**M1**	for $\cot\theta=\frac{1}{\tan\theta}$, $\tan\theta=\frac{\sin\theta}{\cos\theta}$, $\sec\theta=\frac{1}{\cos\theta}$, $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$
	$=\cos\theta$	**A1**	correct simplification, with all correct

	Alternative 2 $\dfrac{\operatorname{cosec}\theta}{\dfrac{1}{\cot\theta}\left(\cot^2\theta+1\right)}$	**M1**	dealing with fractions
	$=\dfrac{\cot\theta\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta}$	**M1**	use of appropriate identity
	$=\dfrac{\cot\theta}{\operatorname{cosec}\theta}$ $=\dfrac{\cos\theta}{\sin\theta}\times\sin\theta$	**M1**	for $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$
	$=\cos\theta$	**A1**	correct simplification, with all correct

9.

4(a)(i)	36	**B1**	
4(a)(ii)	7	**B1**	
4(b)	$[y=]\,5\sin 4x+7$	**B4**	**B1** for each of 5, 4 and 7 and **B1** for sine Accept $a = 5$, $b = 4$, $c = 7$ for **B3**

10.

11(i)	$\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}=\dfrac{\cos^2 x+\sin^2 x}{\sin x\cos x}$ oe	**B2**	**B1** for either $\cot x=\dfrac{\cos x}{\sin x}$ or $\tan x=\dfrac{\sin x}{\cos x}$ used **B1** for correctly placing over a common denominator or for splitting into 3 correct terms **not** just for stating or working from both sides
	Valid use of Pythagorean identity e.g. $\cos^2 x+\sin^2 x=1$	**B1**	
	Simplification to sec$x$ (correct solution only)	**B1**	**not** if working from both sides
11(ii)	$\cos x=\dfrac{1}{2}$ soi	**M1**	
	60, 300	**A1**	Correct pair
	$\cos x=-\dfrac{1}{2}$ soi	**M1**	
	120, 240	**A1**	Correct pair

11.

10(i)	$\sin^{-1}\left(\frac{3}{4}\right)$ soi	**M1**	implied by 0.848[06…]
	0.848[06…] rot to 3 or more figs or 2.29[35…] rot to 3 or more figs	**M1**	implied by a correct answer of acceptable accuracy
	0.544 486... rot to 3 or more figs isw	**A1**	
	1.03 or 1.02630... rot to 4 or more figs isw	**A1**	Maximum 3 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant \frac{\pi}{2}$
10(ii)	Correctly uses $\tan^2 y = \sec^2 y - 1$ and/or $\frac{\sin y}{\cos y}$ **and** $\sin^2 y = 1 - \cos^2 y$	**M1**	for using correct relationship(s) to find an equation in terms of a single trigonometric ratio
	$3\sec^2 y - 14\sec y - 5 = 0$ $\Rightarrow (3\sec y + 1)(\sec y - 5)$ or $5\cos^2 y + 14\cos y - 3 = 0$ $\Rightarrow (5\cos y - 1)(\cos y + 3)$	**DM1**	for factorising or solving their 3-term quadratic dependent on the first M1 being awarded
	[$\cos y = -3$] $\cos y = \frac{1}{5}$	**A1**	
	78.5 or 78.4630... rot to 2 or more decimal places isw	**A1**	
	281.5 or 281.536.... rot to 2 or more decimal places isw	**A1**	Maximum 4 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant 360$

12.

2	$a = 4$	**B1**	
	$b = 6$	**B1**	
	$c = -2$	**M1, A1**	**M1** for use of $\left(\frac{\pi}{12}, 2\right)$ to obtain c, using *their* values of a and of b

13.

Question	Answer	Marks	Guidance
11(a)	$\tan(\phi + 35^\circ) = \frac{2}{5}$	**M1**	dealing correctly with cot and an attempt at solution of $\tan(\phi + 35) = c$, order must be correct, to obtain a value for $\phi + 35$
	$\phi + 35^\circ = 21.8^\circ,\ 201.8^\circ,\ 381.8^\circ$	**M1**	**M1dep** for an attempt at a second solution in the range, (180° + *their* first solution in the range oe)
	$\phi = 166.8^\circ,\ \ 346.8^\circ$	**A2**	**A1** for each
11(b)(i)	**Either** $\dfrac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}}$	**M1**	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ where necessary
	$= \frac{1}{\cos\theta}\left(\frac{\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}\right)$	**M1**	dealing with the fractions correctly to get $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ in denominator or as in left hand column
	$= \frac{\sin\theta}{(1)}$	**A1**	use of identity, together with a complete and correct solution, withhold A1 for incorrect use of brackets
	Or $\dfrac{\sec\theta}{\frac{1}{\tan\theta} + \tan\theta}$ $= \dfrac{\sec\theta}{\frac{1 + \tan^2\theta}{\tan\theta}}$	**M1**	dealing with fractions in the denominator correctly to get $\frac{1 + \tan^2\theta}{\tan\theta}$ in the denominator, allow $\tan\theta$ taken to the numerator
	$= \frac{\sec\theta\tan\theta}{\sec^2\theta}$	**M1**	use of the identity to get $\sec^2\theta$
	$= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$	**A1**	expressing all terms in terms of $\sin\theta$ and $\cos\theta$ and simplification to the given answer, withhold A1 for incorrect use of brackets

Question	Answer	Marks	Guidance
11(b)(ii)	$\sin 3\theta = -\frac{\sqrt{3}}{2}$	**M1**	correct attempt to solve for θ, order must be correct, may be implied by one correct solution
	$3\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}$ $\theta = -\frac{2\pi}{9}, -\frac{\pi}{9}, \frac{4\pi}{9}$	**A3**	**A1** for each

14.

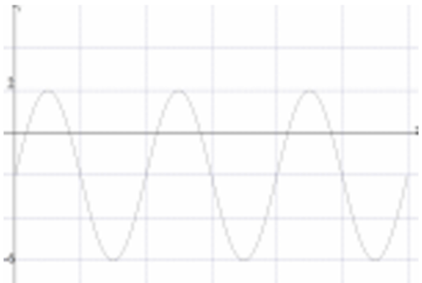
1	Using $\tan^2\theta + 1 = \sec^2\theta$ to obtain $y = 2(\tan^2\theta + 1)$ or $(x+5)^2 = \sec^2\theta - 1$ $(x+5)^2 + 1 = \frac{y}{2}$	**M1**	use of correct identity
	$y = 2((x+5)^2 + 1)$ oe	**A1**	

15.

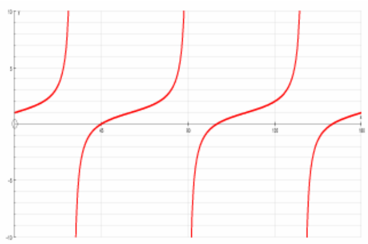
4	$a = 3$	**B1**	
	$b = 8$	**B1**	
	$\frac{5}{2} = 3\cos\left(8 \times \frac{\pi}{12}\right) + c$	**M1**	substitution of $x = \frac{\pi}{12}$ and $y = \frac{5}{2}$ to find c
	$c = 4$	**A1**	

16.

4(i)	$b = 4$	**B1**	
	$c = 6$	**B1**	
	$2 = a + 4\sin\frac{\pi}{2}$	**M1**	Evaluation of a using *their* b and *their* c and the given point.
	$a = -2$	**A1**	

4(ii)		3	**B1** for $-6 \leqslant y \leqslant 2$ **B1** for 3 complete cycles **B1** for all correct

17.

1(i)	$\frac{\pi}{3}$ or 60°	**B1**	
1(ii)		3	**B1** for 3 asymptotes at $x = 30^{\circ}$, 90° and 150°; the curve must approach but not cross all 3 of the asymptotes and be in the 1st and 4th quadrants **B1** for starting at $(0, 1)$ and finishing at $(180, 1)$ **B1** for all correct

18.

8(a)	$3\left(1-\sin^2\theta\right)+4\sin\theta=4$	**M1**	use of correct identity
	$(3\sin\theta-1)(\sin\theta-1)=0$ $\sin\theta=\frac{1}{3},\quad \sin\theta=1$	**M1**	For attempt to solve a 3 term quadratic equation in $\sin\theta$ to obtain $\sin\theta=$
	$\theta=19.5^{\circ},\ 160.5^{\circ}$	**A1**	
	90°	**A1**	
8(b)	$\tan 2\phi=\sqrt{3}$ $2\phi=\frac{\pi}{3},\ -\frac{2\pi}{3}$	**M1**	obtaining an equation in $\tan 2\phi$ and correct attempt to solve for one solution to reach $2\phi=k$
	for one correct solution $\phi=\frac{\pi}{6},$ or 0.524	**A1**	
	for attempt at a second solution	**M1**	
	$\phi=-\frac{\pi}{3}$, or -1.05	**A1**	for a correct second solution and no other solutions within the range

19.

11(a)	$10\left(1-\sin^2 x\right)+3\sin x=9$	**M1**	
	Solves $10\sin^2 x-3\sin x-1=0$ oe	**M1**	**dep** on first **M1** Solves *their* three term quadratic in $\sin x$
	$\sin x=\frac{1}{2},\ \sin x=-\frac{1}{5}$	**A1**	
	30°, 150° and 191.5°, 348.5° awrt	**A2**	**A1** for any two correct solutions
11(b)	$3\frac{\sin 2y}{\cos 2y}=4\sin 2y$ oe	**M1**	
	Solves $3\sin 2y-4\sin 2y\cos 2y\,[=0]$	**M1**	**dep** on first **M1**
	$\sin 2y=0\ \ \cos 2y=\frac{3}{4}$	**A1**	
	Any two of π, 0.72273…, 5.56045… nfww	**A1**	
	$\frac{\pi}{2}$, 0.361, 2.78 awrt nfww	**A1**	**SC**: cancels out sin2*y* after **M1M0** allow **SC1** for 0.72273… and 5.56045... and **SC1** for 0.361 and 2.78

20.

1(i)	Uses $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ $\dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ Uses $\cos^2\theta + \sin^2\theta = 1$ Completes to $\dfrac{1}{\sin\theta} = \operatorname{cosec}\theta$	**B3**	**B1** for using $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ oe or $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ oe at some stage **B1** for use of $\cos^2\theta + \sin^2\theta = 1$ oe **B1** for common denominator of $\sin\theta$ oe either in a compound fraction or in two partial fractions or for writing $\dfrac{1-\sin^2\theta}{\sin\theta}$ as $\dfrac{1}{\sin\theta} - \dfrac{\sin^2\theta}{\sin\theta}$ oe **Maximum of 2 marks if not fully correct or does not complete to cosecθ**
1(ii)	$\sin\theta = \dfrac{1}{4}$	**M1**	
	14.5° or 14.47[751…] rot to 4 or more figures isw	**A1**	Not from wrong working

21.

1	$\sin\left(x + 50^\circ\right) = -\dfrac{1}{\sqrt{2}}$ $\left(x + 50^\circ = -45^\circ,\ 225^\circ\right)$	**M1**	For order of operations – subtraction of 1, division by $\pm\sqrt{2}$ and attempt at $\sin^{-1}$
		M1	**Dep** For obtaining a solution by subtracting 50°
	$x = -95^\circ,\ 175^\circ$	**A2**	**A1** for one correct solution **A1** for a second correct solution and no others within the range

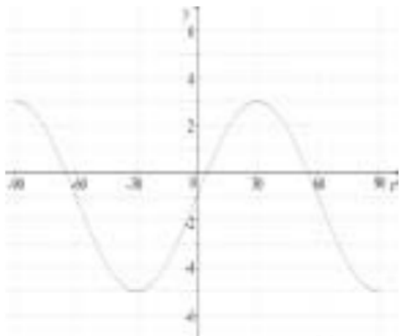
22.

8(i)	$\frac{(1+\sin x)-(1-\sin x)}{(1-\sin x)(1+\sin x)}$	**M1**	
	$\frac{2\sin x}{1-\sin^2 x}$	**A1**	
	$\frac{2\sin x}{\cos^2 x}$	**M1**	
	$\frac{2\sin x}{\cos x}\times\frac{1}{\cos x}=2\tan x\sec x$	**A1**	**AG**
8(ii)		**M1**	equate $2\sec x\tan x=\text{cosec}\,x$
	$\tan^2 x=\frac{1}{2}$	**A1**	
	$35.3°, 144.7°, 215.3°, 324.7°$	**2**	**A1** two correct

23.

9(a)	$x+\frac{\pi}{4}=\frac{\pi}{3}$	**M1**	
	$\frac{\pi}{12}$ and $\frac{5\pi}{12}$ (0.262 and 1.31)	**A2**	**A1** for one correct
9(b)	correctly use $\sec y=\frac{1}{\cos y}$ and $\text{cosec}\,y=\frac{1}{\sin y}$	**M1**	
	$\tan y=\frac{4}{3}$	**A1**	obtain expression for tan$y$ or y explicitly
	53.1° and 233.1°	**A1**	
9(c)	correctly rewrite equation in terms of sin$z$ and cos$z$	**M1**	
	use $\sin^2 z=1-\cos^2 z$	**M1**	appropriate use of pythagorean identity for forming an equation in one trig ratio
	$8\cos^2 z-2\cos z-1=0$ oe	**A1**	
	$(4\cos z+1)(2\cos z-1)=0$	**M1**	solve 3 term quadratic in cos$z$
	60° and 300° and 104.5° and 255.5°	**A2**	**A1** for any two correct

24.

2(i)	4	**B1**	
2(ii)	120° or $\dfrac{2\pi}{3}$	**B1**	
2(iii)		**B3**	**B1** for a complete curve starting at $(-90^\circ, 3)$ and finishing at $(90^\circ, -5)$ **B1** for $-5 \leqslant y \leqslant 3$ for a complete curve Minimum point(s) at $y = -5$ Maximum point(s) at $y = 3$ **DepB1** for a fully correct sine curve satisfying both the above and passing through $(-60^\circ, -1), (0^\circ, -1)$ and $(60^\circ, -1)$

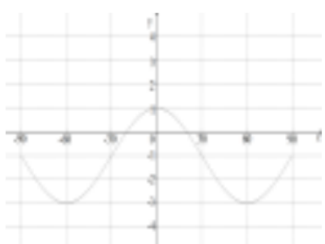
25.

11(a)(i)	$\dfrac{1}{\sin\theta}\left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)$	**M2**	**M1** for either $\dfrac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \dfrac{1}{\sin\theta}\left(\operatorname{cosec}\theta - \dfrac{\cos\theta}{\sin\theta}\right)$ or $\dfrac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \dfrac{1}{\sin\theta}\left(\dfrac{1}{\sin\theta} - \cot\theta\right)$
	$\dfrac{1-\cos\theta}{1-\cos^2\theta}$	**M1**	
	$\dfrac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)} = \dfrac{1}{1+\cos\theta}$	**A1**	
11(a)(ii)	awrt 233.1	**B2**	with no extras in range **B1** for $\cos\theta = -\dfrac{3}{5}$ soi
11(b)	$3\phi - 4 = \tan^{-1}\left(-\dfrac{1}{2}\right)$ soi	**M1**	
	awrt 0.132, 1.18	**A2**	with no extras in range **A1** for one correct

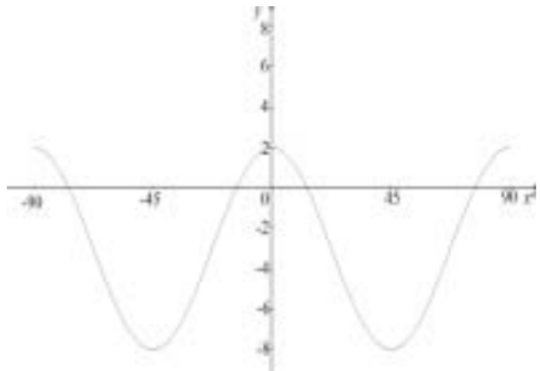
26.

9(a)	$6(1-\cos^2 x)-13\cos x = 1$ oe	**B1**	
	Solves or factorises *their* 3-term quadratic	**M1**	
	70.5 and 289.5	**A2**	with no extras in range **A1** for either, ignoring extras in range
9(b)(i)	Numerator: Substitution of $\tan y = \frac{\sin y}{\cos y}$	**M1**	
	Denominator: Substitution of $1+\tan^2 y = \sec^2 y$ or substitution of $1+\tan^2 y = 1+\frac{\sin^2 y}{\cos^2 y}$ **and** correct rearrangement to $\frac{1}{\cos^2 y}$ oe	**M1**	
	Correct completion to $4\sin y$ cao	**A1**	
9(b)(ii)	–0.848[06…] rot to 3 or more figures	**B1**	with no extras in range

27.

1(i)		**B3**	**B1** for y intercept $(0,1)$, must have a graph **B1** for starting and finishing at $(\pm 90,-1)$ **B1** for all correct, must be attempt at a curve passing through $(\pm 30,-1)$ and $(\pm 60,-3)$
1(ii)	2	**B1**	
1(iii)	$120°$ or $\frac{2\pi}{3}$	**B1**	

28.

2(i)		**B4**	**B1** for a maximum at $(0, 2)$ **B1** for minimums at $y = -8$ and no other minimums **B1** for starting at $(-90^\circ, 2)$ and finishing at $(90^\circ, 2)$ **B1** for a fully correct curve with correct shape, particularly at end points, that has earned all three previous B marks.
2(ii)	5	**B1**	
2(iii)	90°	**B1**	

29.

Question	Answer	Marks	Guidance
6(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	**M1**	Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	**M1**	**dep** Multiply by $\cos x$
	$\dfrac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{(1+\cos x)\sin x}$	**M1**	**dep** Add *their* fractions correctly and expand $(1 + \cos x)^2$ correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	**M1**	**dep** Use $\sin^2 x + \cos^2 x = 1$ and take out a factor of 2.
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.

	OR $\dfrac{\tan^2 x + (\sec x + 1)^2}{\tan x(\sec x + 1)}$	**M1**	Add fractions
	$= \dfrac{2\sec^2 x + 2\sec x}{\tan x(\sec x + 1)}$	**M1**	**dep** Expand brackets correctly and use $1 + \tan^2 x = \sec^2 x$
	$\dfrac{2\sec x}{\tan x}$	**M1**	**dep** Cancel $\sec x + 1$
	$\dfrac{2}{\cos x} \times \dfrac{\cos x}{\sin x}$	**M1**	**dep** Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ oe
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.

6(ii)	$3\sin^2 x + \sin x - 2 = 0$ oe	**B1**	Obtain three term quadratic.
	$(3\sin x - 2)(\sin x + 1) = 0$	**M1**	Solve three term quadratic
	41.8° awrt	**A1**	
	138.2° awrt	**A1**	Mark final answers This mark is not awarded if there are more solutions in the range.

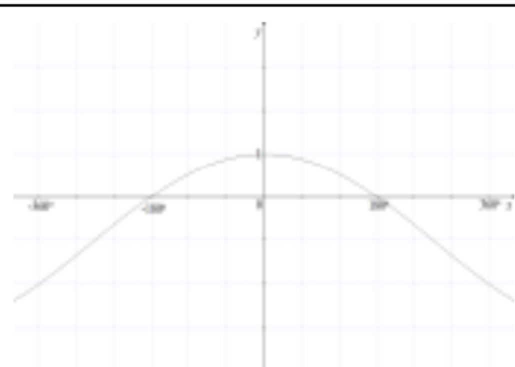
30.

2(i)	$\dfrac{\frac{1}{\sin} - \frac{\cos x}{\sin x}}{1 - \cos x}$	**M1**	express in terms of $\sin x$ and $\cos x$
	$\dfrac{(1 - \cos x)}{\sin(1 - \cos x)}$	**A1**	rewrite not as a fraction within a fraction
	$\dfrac{1}{\sin x} = \operatorname{cosec} x$	**A1**	correct completion **answer given**
2(ii)	$\left[\sin x = \dfrac{1}{2}\right]$ $x = 30°$	**B1**	
	$x = 150°$ nfww	**B1**	no extra answers

31.

5(a)	$\tan\left(y-\frac{\pi}{4}\right)=(\pm)\sqrt{3}$	**M1**	$\pm$ 1.73…
	$y-\frac{\pi}{4}=\frac{\pi}{3}$ or $\frac{2\pi}{3}$	**A1**	1.04(7…) or 2.09(4…)
	$y=\frac{7\pi}{12}$ or 1.83	**A1**	
	$y=\frac{11\pi}{12}$ or 2.88	**A1**	
5(b)	correctly rewrite equation in terms of sin$z$ and cos$z$	**M1**	
	use $\sin^2 z = 1 - \cos^2 z$	**M1**	appropriate use of Pythagorean identity for forming an equation in one trig ratio
	$6\cos^2 z - 7\cos z + 1 = 0$ **oe**	**A1**	
	$(6\cos z - 1)(\cos z - 1) = 0$	**M1**	solve three term quadratic in cos$z$
	80.4°	**A1**	
	279.6°	**A1**	

32.

2(a)	1080°	**B1**	
2(b)		**B1**	For correct shape and symmetry about the y-axis
		B1	For correct x-intercepts
		B1	For correct y-intercept

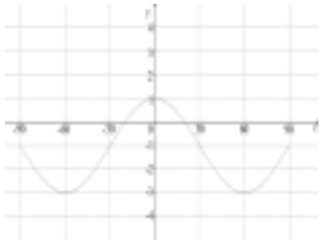
33.

9(a)	$x+\frac{\pi}{4}=\frac{\pi}{3}$	**M1**	
	$\frac{\pi}{12}$ and $\frac{5\pi}{12}$ (0.262 and 1.31)	**A2**	**A1** for one correct
9(b)	correctly use $\sec y=\frac{1}{\cos y}$ and $\operatorname{cosec} y=\frac{1}{\sin y}$	**M1**	
	$\tan y=\frac{4}{3}$	**A1**	obtain expression for tan*y* or *y* explicitly
	53.1° and 233.1°	**A1**	
9(c)	correctly rewrite equation in terms of sin*z* and cos*z*	**M1**	
	use $\sin^2 z=1-\cos^2 z$	**M1**	appropriate use of pythagorean identity for forming an equation in one trig ratio
	$8\cos^2 z-2\cos z-1=0$ oe	**A1**	
	$(4\cos z+1)(2\cos z-1)=0$	**M1**	solve 3 term quadratic in cos*z*
	60° and 300° and 104.5° and 255.5°	**A2**	**A1** for any two correct

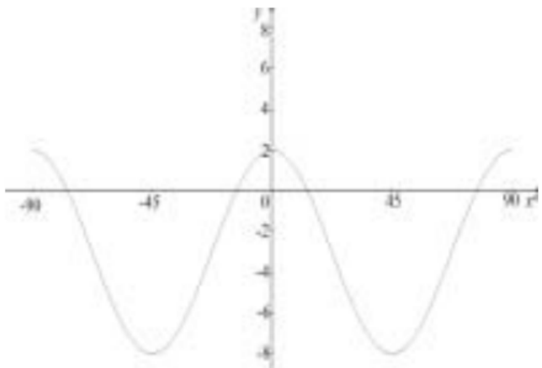
34.

9(a)	$6(1-\cos^2 x)-13\cos x=1$ oe	**B1**	
	Solves or factorises *their* 3-term quadratic	**M1**	
	70.5 and 289.5	**A2**	with no extras in range **A1** for either, ignoring extras in range
9(b)(i)	Numerator: Substitution of $\tan y=\frac{\sin y}{\cos y}$	**M1**	
	Denominator: Substitution of $1+\tan^2 y=\sec^2 y$ or substitution of $1+\tan^2 y=1+\frac{\sin^2 y}{\cos^2 y}$ **and** correct rearrangement to $\frac{1}{\cos^2 y}$ oe	**M1**	
	Correct completion to $4\sin y$ cao	**A1**	
9(b)(ii)	–0.848[06…] rot to 3 or more figures	**B1**	with no extras in range

35.

1(i)		**B3**	**B1** for y intercept $(0,1)$, must have a graph **B1** for starting and finishing at $(\pm 90, -1)$ **B1** for all correct, must be attempt at a curve passing through $(\pm 30, -1)$ and $(\pm 60, -3)$
1(ii)	2	**B1**	
1(iii)	120° or $\frac{2\pi}{3}$	**B1**	

36.

2(i)		**B4**	**B1** for a maximum at $(0, 2)$ **B1** for minimums at $y = -8$ and no other minimums **B1** for starting at $(-90^\circ, 2)$ and finishing at $(90^\circ, 2)$ **B1** for a fully correct curve with correct shape, particularly at end points, that has earned all three previous B marks.
2(ii)	5	**B1**	
2(iii)	90°	**B1**	

37.

Question	Answer	Marks	Guidance
6(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	M1	Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	M1	**dep** Multiply by $\cos x$
	$\dfrac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$	M1	**dep** Add *their* fractions correctly and expand $(1 + \cos x)^2$ correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	M1	**dep** Use $\sin^2 x + \cos^2 x = 1$ and take out a factor of 2.
	All correct AG	A1	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
	OR $\dfrac{\tan^2 x+(\sec x+1)^2}{\tan x(\sec x+1)}$	M1	Add fractions
	$=\dfrac{2\sec^2 x+2\sec x}{\tan x(\sec x+1)}$	M1	**dep** Expand brackets correctly and use $1 + \tan^2 x = \sec^2 x$
	$\dfrac{2\sec x}{\tan x}$	M1	**dep** Cancel $\sec x + 1$
	$\dfrac{2}{\cos x}\times\dfrac{\cos x}{\sin x}$	M1	**dep** Use $\tan x = \dfrac{\sin x}{\cos x}$ and $\sec x = \dfrac{1}{\cos x}$ oe
	All correct AG	A1	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
6(ii)	$3\sin^2 x + \sin x - 2 = 0$ oe	B1	Obtain three term quadratic.
	$(3\sin x - 2)(\sin x + 1) = 0$	M1	Solve three term quadratic
	41.8° awrt	A1	
	138.2° awrt	A1	Mark final answers This mark is not awarded if there are more solutions in the range.

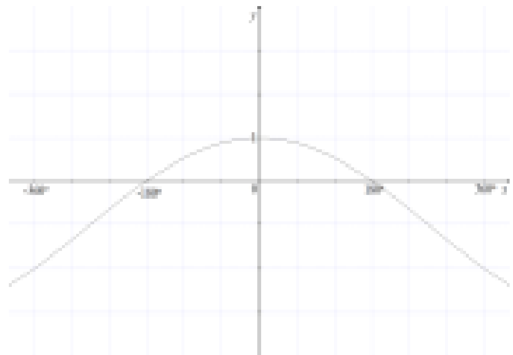
38.

2(i)	$\dfrac{\frac{1}{\sin} - \frac{\cos x}{\sin x}}{1-\cos x}$	**M1**	express in terms of sin$x$ and cos$x$
	$\dfrac{(1-\cos x)}{\sin(1-\cos x)}$	**A1**	rewrite not as a fraction within a fraction
	$\dfrac{1}{\sin x} = \operatorname{cosec} x$	**A1**	correct completion **answer given**
2(ii)	$\left[\sin x = \dfrac{1}{2}\right]$ $x = 30°$	**B1**	
	$x = 150°$ nfww	**B1**	no extra answers

39.

5(a)	$\tan\left(y - \dfrac{\pi}{4}\right) = (\pm)\sqrt{3}$	**M1**	± 1.73…
	$y - \dfrac{\pi}{4} = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$	**A1**	1.04(7…) or 2.09(4…)
	$y = \dfrac{7\pi}{12}$ or 1.83	**A1**	
	$y = \dfrac{11\pi}{12}$ or 2.88	**A1**	
5(b)	correctly rewrite equation in terms of sin$z$ and cos$z$	**M1**	
	use $\sin^2 z = 1 - \cos^2 z$	**M1**	appropriate use of Pythagorean identity for forming an equation in one trig ratio
	$6\cos^2 z - 7\cos z + 1 = 0$ **oe**	**A1**	
	$(6\cos z - 1)(\cos z - 1) = 0$	**M1**	solve three term quadratic in cos$z$
	80.4°	**A1**	
	279.6°	**A1**	

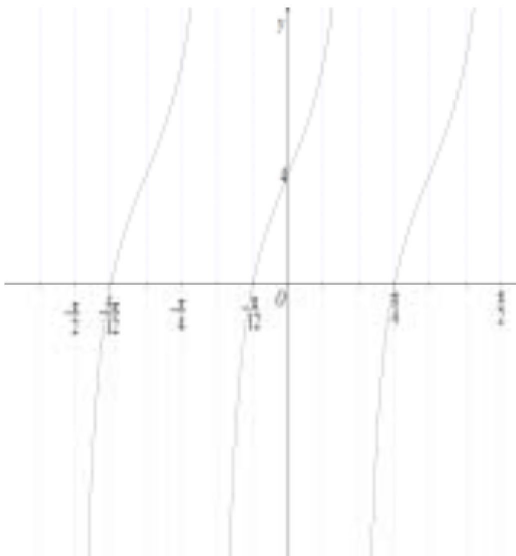
40.

2(a)	$1080°$	**B1**	
2(b)		**B1**	For correct shape and symmetry about the y-axis
		B1	For correct x-intercepts
		B1	For correct y-intercept

41.

10(a)(i)	$\dfrac{(\sec\theta+1)-(\sec\theta-1)}{\sec^2\theta-1}$	**M1**	For dealing with the fractions
	$\dfrac{2}{\tan^2\theta}$	**M1**	For use of the correct identity
	$2\cot^2\theta$	**A1**	A1 for given answer, must see $\dfrac{8}{\tan^2\theta}$ first
10(a)(ii)	$2\cot^2 2x=6$ $\tan 2x=\pm\dfrac{1}{\sqrt{3}}$	**M1**	M1 for use of (i) and attempt to simplify
		A1	
		M1	M1 for attempt to solve, may be implied by one correct solution
	$2x=-150°,\ -30°,\ 30°,\ 150°$ $x=-75°,\ -15°,\ 15°,\ 75°$	**A2**	A1 for each pair of correct solutions
10(b)	$\sin\left(y+\dfrac{\pi}{3}\right)=\dfrac{1}{2}$	**M1**	For dealing with cosec and an attempt to solve
	$y+\dfrac{\pi}{3}=\dfrac{5\pi}{6},\ \dfrac{13\pi}{6}$	**M1**	M1 for a complete method of solution, may be implied by a correct solution
	$y=\dfrac{\pi}{2}$	**A1**	
	$y=\dfrac{11\pi}{6}$	**A1**	

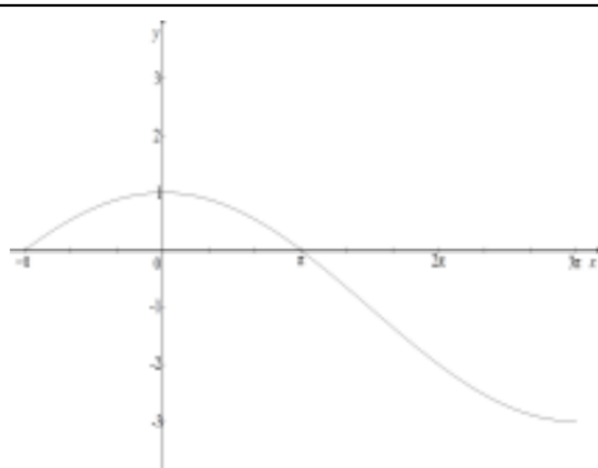
42.

Question	Answer	Marks	Partial Marks
10(a)	$3x = -\frac{5\pi}{4} - \frac{\pi}{4}, \frac{3\pi}{4}$	**M1**	For a correct attempt to solve, may be implied by one correct solution
	$x = -\frac{\pi}{12}$	**A1**	
	$x = \frac{\pi}{4}$	**A1**	
	$x = -\frac{5\pi}{12}$	**A1**	
10(b)		**B1**	Shape – must have three 'parts' with asymptotes
		B1	For correct x-coordinates
		B1	For correct y-coordinate

43.

8(a)	$3(\text{cosec}^2 x - 1) - 14\,\text{cosec}\,x - 2[=0]$	**M1**	
	$3\text{cosec}^2 x - 14\text{cosec}x - 5 = 0$	**A1**	
	$(\text{cosec}x - 5)(3\text{cosec}x + 1)$	**M1**	
	$\sin x = \frac{1}{5}$ nfww	**A1**	
	11.5 and 168.5 nfww	**A1**	
8(b)	Correct use of $\sin^2 y + \cos^2 y = 1$	**B1**	
	Factorises using the difference of 2 squares	**B1**	
	Uses $\frac{1}{\cot y} = \tan y$ or $\cot y = \frac{\cos y}{\sin y}$ correctly	**B1**	
	Full and correct completion to given answer: $\tan y - 2\cos y \sin y$	**B1**	

44.

3(a)	2	**B1**	
3(b)	6π or $1080°$	**B1**	
3(c)		3	**B1** for passing through $(-\pi, 0)$ and $(3\pi, -3)$ – must be a curve **B1** for correct shape with max on y-axis and a min at $x = 3\pi$ **B1** for passing through $(0, 1)$ and $(\pi, 0)$ only on the positive x-axis

45.

11(a)	$2\cos x = 3\dfrac{\sin x}{\cos x} \Rightarrow 2\cos^2 x = 3\sin x$	**M1**	For use of $\tan x = \dfrac{\sin x}{\cos x}$ and multiplying by $\cos x$
	$2(1-\sin^2 x) = 3\sin x$	**M1**	For use of correct identity
	$2\sin^2 x + 3\sin x - 2 = 0$	**A1**	For correct rearrangement to obtain the given answer
	Alternative $2\sin^2 x + 3\sin x - 2$ $= 2(1-\cos^2 x) + 3\sin x - 2$	**(M1)**	For use of correct identity
	$= -2\cos x\cos x + 3\sin x$ $= -3\tan x\cos x + 3\sin x$	**(M1)**	For use of $2\cos x = 3\tan x$
	$-3\sin x + 3\sin x = 0$	**(A1)**	For use of $\tan x\cos x = \sin x$ and answer 0

Question	Answer	Marks	Guidance
11(b)	$\sin\left(2\alpha + \dfrac{\pi}{4}\right) = \dfrac{1}{2}$ only	**B1**	For solution of quadratic from **(a)** to obtain $\sin\left(2\alpha + \dfrac{\pi}{4}\right) = \dfrac{1}{2}$ only
	$2\alpha + \dfrac{\pi}{4} = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}$ $2\alpha = \dfrac{7\pi}{12}, \dfrac{23\pi}{12}$	**M1**	For correct order of operations in attempt to solve $\sin\left(2\alpha + \dfrac{\pi}{4}\right) = \dfrac{1}{2}$, may be implied by one correct solution
	$\alpha = \dfrac{7\pi}{24}$	**A1**	
	$\alpha = \dfrac{23\pi}{24}$	**A1**	

46.

11(a)	$\text{LHS} = \dfrac{\sin x \times \dfrac{\sin x}{\cos x}}{1 - \cos x}$	**M1**	Uses $\tan x = \dfrac{\sin x}{\cos x}$
	$= \dfrac{1 - \cos^2 x}{\cos x(1 - \cos x)}$	**M1**	**Dep** Uses $\sin^2 x = 1 - \cos^2 x$ to eliminate $\sin x$
	$\dfrac{(1 - \cos x)(1 + \cos x)}{\cos x(1 - \cos x)} = \dfrac{1 + \cos x}{\cos x} = \sec x + 1$	**2**	**M1Dep** Factorise correctly and cancel correctly. **A1** Uses $\dfrac{1}{\cos x} = \sec x$
11(b)	$5\dfrac{\sin x}{\cos x} - 3\dfrac{\cos x}{\sin x} = \dfrac{2}{\cos x}$	**B1**	Change $\tan x$, $\cot x$ and $\sec x$ into $\sin x$ and $\cos x$ correctly.
	$5\sin^2 x - 3(1 - \sin^2 x) = 2\sin x$	**M1**	Multiply correctly by $\sin x \cos x$ and use $\cos^2 x + \sin^2 x = 1$
	$8\sin^2 x - 2\sin x - 3 = 0$	**A1**	Three term quadratic.
	$(2\sin x + 1)(4\sin x - 3) = 0$	**M1**	Factorise or use formula on *their* quadratic
	$\sin x = -\dfrac{1}{2} \rightarrow x = 210°, 330°$	**A1**	
	$\sin x = \dfrac{3}{4} \rightarrow x = 48.6°, 131.4°$	**A1**	

47.

9(a)(i)	$(3y+2)(2x+1)$	B1	
9(a)(ii)	$(3\cos\theta+2)(2\sin\theta+1)=0$ $\cos\theta=-\frac{2}{3}$, $\sin\theta=-\frac{1}{2}$	M1	For relating to part (i) and a correct attempt to obtain $\cos\theta=...$ or $\sin\theta=...$
	$\theta=131.8°, 228.2°$ $\theta=210°, 330°$	3	**M1** for solving one of the equations to obtain one correct solution **A1** for any two correct solutions **A1** for a further two correct solutions with no extra solutions within the range
9(b)	$\cos\left(2\phi+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$ oe	B1	
	$\phi=-\frac{5\pi}{24},-\frac{\pi}{24},\frac{19\pi}{24},\frac{23\pi}{24}$	4	**M1** for solving to obtain one correct positive solution **M1** for solving to obtain one correct negative solution **A1** for any two correct solutions **A1** for a further two correct solutions with no extra solutions within the range

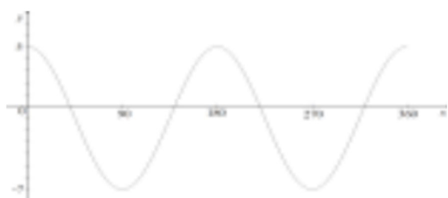
48.

10(a)	$\frac{1}{\sin\alpha}+\frac{1}{\cos\alpha}$ $(=0)$	B1	For dealing correctly with $\mathrm{cosec}^2\alpha$ and $\sec^2\alpha$ to obtain an expression in $\sin\alpha$ and $\cos\alpha$ only
	$\tan\alpha=-1$ or $\sin\alpha=-\cos\alpha$	B1	For an equation in $\tan\alpha$, may be implied by a correct solution.
	$\alpha=-\frac{\pi}{4}$ or -0.785 $\alpha=\frac{3\pi}{4}$ or 2.36	2	**B1** for one correct solution **B1** for a second correct solution and no extra solutions in the range.

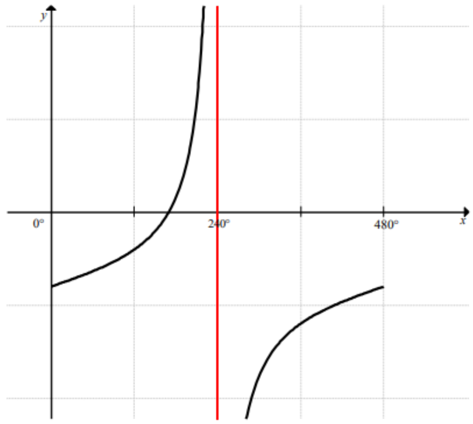
Question	Answer	Marks	Guidance
10(b)(i)	$\dfrac{\cos^2\theta+1-2\sin\theta+\sin^2\theta}{\cos\theta(1-\sin\theta)}$	**M1**	For dealing with the fractions correctly and expansion of $(1-\sin\theta)^2$
	$\dfrac{1+1-2\sin\theta}{\cos\theta(1-\sin\theta)}$ or better	**M1**	**Dep** for use of identity, may be implied by $\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$
	$\dfrac{2(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$	**M1**	**Dep** on previous M mark for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**A1**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 1 $\left(\dfrac{\cos\theta}{1-\sin\theta}\times\dfrac{1+\sin\theta}{1+\sin\theta}\right)+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	
	$\dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** for use of identity
	$\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** on previous M mark for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**(A1)**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.
	Alternative 2 $\dfrac{(1-\sin^2\theta)+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$	**(M1)**	For dealing with the fractions and using $\cos^2\theta=1-\sin^2\theta$.
	$\dfrac{(1-\sin\theta)(1+\sin\theta)+(1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$	**(M1)**	**Dep** for factorising $1-\sin^2\theta$
	$\dfrac{1+\sin\theta+1-\sin\theta}{\cos\theta}$	**(M1)**	**Dep** for simplification
	$\dfrac{2}{\cos\theta}=2\sec\theta$	**(A1)**	Need to see this detail for A1 Need to have had θ in every trigonometric ratio.

Question	Answer	Marks	Guidance
10(b)(ii)	$\cos 3\phi = \frac{1}{2}$	B1	
	$\phi = 20^\circ, 100^\circ, 140^\circ$	3	**M1** for one correct solution of *their* $\cos 3\phi = k$ using a correct order of operations **A1** for 2 correct solutions **A1** for a third correct solution with no extra solutions in the range

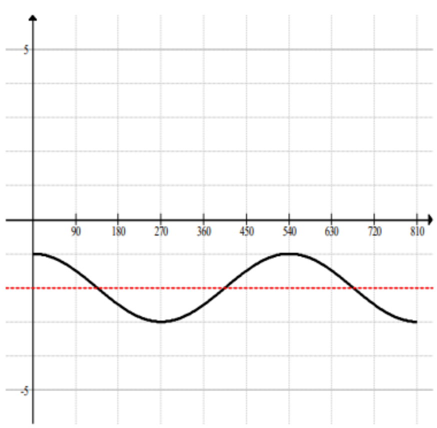
49.

1(a)		3	**B1** for shape, must have implied symmetry about $x = 180^\circ$, 1 complete maximum point above the x-axis, 2 minimum points below the x-axis, starting and finishing at the same positive y value. 5 and – 7 not necessary for this mark. **B1** for $-7 \leqslant y \leqslant 5$, may be implied by numbers in a table if not seen on the graph or by coordinates. Must have maximum point(s) at $y = 5$ only and minimum point(s) at $y = -7$ only. **B1** for a completely correct curve with maximum points implied at the end points.
1(b)	$a = 0$ $b = 4$ $c = 1$	2	**B1** for 2 correct.
	Alternative $a = 0$ $b = -4$ $c = -1$	(2)	**B1** for 2 correct.

50.

4(a)	$a = -4$	**B1**	
	$480 = \dfrac{180}{b}$ oe	**M1**	
	$b = \dfrac{3}{8}$	**A1**	
4(b)	Correct sketch 	**B2**	correct tan shape, two branches starting and finishing on same negative y value asymptote implied at $x = 240$ root between 120 and 240 **B1** for correct tan shape with exactly two branches plus one other correct property Maximum **B1** if not fully correct

51.

5(a)	1	**B1**	
5(b)	$360 \div \dfrac{2}{3}$ oe	**M1**	
	540	**A1**	If 0 scored, **SC1** for 3π
5(c)	Correct sketch for domain $0° \leqslant x \leqslant 810°$ 	**B2**	**B1** for correct cosine shape from $(0, -1)$ with amplitude 1 for $0° \leqslant x \leqslant 810°$ **B1** for attempt at correct cosine shape with period 540° for $0° \leqslant x \leqslant 810°$ If 0 scored, **SC1** for a fully correct graph for $0° \leqslant x \leqslant 540°$ **Maximum of 1 mark if not fully correct.**

52.

8(a)(i)	Uses correct Pythagorean identity in the left-hand side of the given identity, e.g. $\frac{1-\sin^2 2x}{1+\sin 2x}$	**M1**	
	$\frac{(1-\sin 2x)(1+\sin 2x)}{1+\sin 2x}$ oe and completion to given answer	**A1**	
8(a)(ii)	$\sin 2x = \frac{2}{3}$	**M1**	
	$x = \frac{1}{2}\sin^{-1}\left(\frac{2}{3}\right)$ soi	**M1**	dep on first **M1**
	20.9 or 20.905… rounded or truncated to 4 or more figures and 69.1 or 69.094… rounded or truncated to 4 or more figures	**A2**	with no incorrect values in range **A1** for either angle correct, ignoring extra values
8(b)	$\tan\left(y - \frac{\pi}{2}\right) = \frac{1}{\sqrt{3}}$ soi	**M1**	
	$y = \frac{\pi}{6} + \frac{\pi}{2}$	**M1**	dep on first **M1**
	$\frac{2}{3}\pi$ oe or 2.09 or 2.094[39…] rot to 4 or more significant figs	**A1**	with no incorrect values in range

53.

4	$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$	**B1**	
	$x = -\frac{7\pi}{12},\ -\frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{11\pi}{12}$	**3**	**M1** for using correct order of operations **A1** for two correct solutions **A1** for two further correct solutions and no other solutions in range

54.

3	$\tan\left(2x - \frac{\pi}{3}\right) = \pm\sqrt{3}$ soi or $\sin\left(2x - \frac{\pi}{3}\right) = \pm\frac{\sqrt{3}}{2}$ soi	B1	**B0** if negative root is rejected Allow truncated decimals May be implied by subsequent work From use of $\text{cosec}^2\left(2x - \frac{\pi}{3}\right) - 1 = \cot^2\left(2x - \frac{\pi}{3}\right)$
	$2x - \frac{\pi}{3} = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ $2x = 0, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}$ $x = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{6}$ or 0, 1.05, 1.57, 2.62 or greater accuracy	4	**M1** for correct order of operations to obtain one solution in the range using $\tan\left(2x - \frac{\pi}{3}\right) = k$ or $\sin\left(2x - \frac{\pi}{3}\right) = m$, $\lvert m \rvert < 1$ **Dep M1** for correct order of operations to obtain a second solution in the range using $\left(2x - \frac{\pi}{3}\right) = \tan^{-1}(k) \pm \pi$ or $\left(2x - \frac{\pi}{3}\right) = \pi - \sin^{-1}(m)$, $\lvert m \rvert < 1$ oe $\left(2x - \frac{\pi}{3}\right) = -\sin^{-1}(m)$, $\lvert m \rvert < 1$ oe **A1** for any pair of correct solutions **A1** for remaining pair of solutions, with no extra solutions within the range

55.

3(a)	$\frac{\cos x}{1-\cos x}+\frac{\cos x}{1+\cos x}$ or $\frac{\sec x+1+\sec x-1}{\sec^2 x-1}$	**M1**	
	$\frac{\cos x+\cos^2 x+\cos x-\cos^2 x}{1-\cos^2 x}$ or $\frac{2\sec x}{\tan^2 x}$	**A1**	
	$\frac{2\cos x}{\sin^2 x}$ or $\frac{2\cos^2 x}{\cos x\sin^2 x}$ oe	**A1**	
	Fully correct justification of given answer: $2\cot x\operatorname{cosec} x$	**A1**	
3(b)	$3\tan^2 x=2$ oe or better, soi or $5\cos^2 x=3$ oe or better, soi or $5\sin^2 x=2$ oe or better, soi	**B1**	
	$\tan x=[\pm]\sqrt{\frac{2}{3}}$ oe or [±] 0.816[4…] or $\cos x=[\pm]\sqrt{\frac{3}{5}}$ oe or [±] 0.774[5…] or $\sin x=[\pm]\sqrt{\frac{2}{5}}$ oe or [±] 0.632[4…]	**M1**	**FT** an equation of the form $a\tan^2 x=b$, $a>0$, $b>0$ or $p\sin^2 x=q$ or $p\cos^2 x=q$ where $p>0$, $q>0$ and $p>q$
	39.2° or 39.2315… rot to 2 or more dp 140.8° or 140.7684… rot to 2 or more dp 219.2° or 219.2315… rot to 2 or more dp 320.8° or 320.7684… rot to 2 or more dp	**A2**	no extras in range **A1** for any two correct answers

56.

3(a)	$a=3$	**B1**	
	$b=2$	**B1**	
	$c=-1$	**B1**	
3(b)(i)	2	**B1**	
3(b)(ii)	$\frac{2\pi}{3}$ oe or 2.09 or 2.094[395…] rot to 4 or more sf	**B1**	

57.

5(a)	$\frac{\sin x}{1-\sin x}+\frac{\sin x}{1+\sin x}$ or $\frac{\operatorname{cosec} x+1+\operatorname{cosec} x-1}{\operatorname{cosec}^2 x-1}$ oe	**M1**	
	$\frac{\sin x+\sin^2 x+\sin x-\sin^2 x}{1-\sin^2 x}$ or $\frac{2\operatorname{cosec} x}{\cot^2 x}$ oe	**A1**	
	$\frac{2\sin x}{\cos^2 x}$ or $\frac{2\sin^2 x}{\sin x\cos^2 x}$ oe	**A1**	
	Fully correct justification of given answer: $\frac{2\sin x}{\cos x}\times\frac{1}{\cos x}=2\tan x\sec x$ or $2\tan x\times\frac{1}{\cos x}=2\tan x\sec x$ or $\frac{2\sin x}{\cos x}\times\sec x=2\tan x\sec x$ or equivalent	**A1**	
5(b)	$2\tan^2 x=5$ or better, soi or $7\cos^2 x=2$ or better, soi or $7\sin^2 x=5$ or better, soi	**B1**	
	$\tan x=[\pm]\sqrt{\frac{5}{2}}$ oe or [±] 1.58[1…] or $\cos x=[\pm]\sqrt{\frac{2}{7}}$ oe or [±] 0.534[5…] or $\sin x=[\pm]\sqrt{\frac{5}{7}}$ oe or [±] 0.845[1…]	**M1**	**FT** an equation of the form $a\tan^2 x=b$ $a>0, b>0$ or $p\sin^2 x=q$ or $p\cos^2 x=q$ where $p>0$, $q>0$ and $p>q$
	57.7 or 57.6884… rot to 2 or more dp 237.7 or 237.6884… rot to 2 or more dp 122.3 or 122.3115… rot to 2 or more dp 302.3 or 302.3115… rot to 2 or more dp	**A2**	no extras in range **A1** for any two correct answers

58.

6(a)	$k = 14$	**B1**	
	$k = 6$	**B1**	
6(b)(i)	$\dfrac{(1+\tan\theta)(1+\cos\theta)+(1-\tan\theta)(1-\cos\theta)}{1-\cos^2\theta}$	**M1**	Allow $(1 + \cos\theta)(1 - \cos\theta)$ in the denominator
	Expansion of numerator and simplification of denominator	**M1**	**Dep** on previous M mark
	Use of $\tan\theta\cos\theta = \sin\theta$	**B1**	soi
	$\dfrac{2(1+\sin\theta)}{\sin^2\theta}$	**A1**	Sufficient simplification to justify obtaining the given answer
6(b)(ii)	$2(1 + \sin\theta) = 3\sin^2\theta$ $3\sin^2\theta - 2\sin\theta - 2 = 0$	**M1**	For use of part **(a)** and attempt to simplify to a 3-term quadratic equation equated to zero.
	$\sin\theta = \dfrac{1-\sqrt{7}}{3}$ or $-0.5485\ldots$	**M1**	**M1** for attempt to solve and obtain a value for θ, may be implied by one correct solution
	$213.3°$ and $326.7°$	**A2**	**A1** for one solution If 0 scored **SC1** for awrt 213 and 327 Penalise excess solutions in the range

59.

8(a)	$x^2 + 2\sqrt{5}x - 20 = 3\sqrt{5}x + 10$ $x^2 - \sqrt{5}x - 30 = 0$	**M1**	For equating x terms and simplifying to a 3-term quadratic equation equated to zero.
	$x = \dfrac{\sqrt{5} \pm \sqrt{5-(4\times -30)}}{2}$ oe	**M1**	**Dep** on previous M mark for attempt to solve to obtain $x =$, sufficient detail must be shown
	$x = 3\sqrt{5}$ $\quad x = -2\sqrt{5}$	**A1**	For both
	$y = 55, y = -20$	**A1**	For both

8(b)	Use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	**B1**	May be implied by later work
	$\operatorname{cosec}^2\theta = 1 + \dfrac{\left(2+\sqrt{3}\right)^2}{\left(\sqrt{3}-1\right)^2}$	**M1**	For attempting to deal with tan correctly, forming a single fraction and simplifying, with sufficient detail – at least 4 terms in the numerator
	$\operatorname{cosec}^2\theta = \dfrac{11+2\sqrt{3}}{4-2\sqrt{3}}$	**A1**	
	$\operatorname{cosec}^2\theta = \dfrac{11+2\sqrt{3}}{4-2\sqrt{3}} \times \dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$	**M1**	For attempt to rationalise *their* expression, with sufficient detail in the simplification of the numerator – at least 3 terms
	$14 + \dfrac{15\sqrt{3}}{2}$	**A1**	
	Alternative 1 Use of $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$	**(B1)**	 May be implied by later work
	$\cot\theta = \dfrac{2+\sqrt{3}}{\sqrt{3}-1}$ $= \dfrac{3\sqrt{3}+5}{2}$	**(2)**	**M1** for attempting to rationalise $\cot\theta$ or tan with sufficient detail in the simplification of the numerator – at least 3 terms
	$\operatorname{cosec}^2\theta = 1 + \left(\dfrac{3\sqrt{3}+5}{2}\right)^2$ $= 14 + \dfrac{15\sqrt{3}}{2}$	**(2)**	**M1** for expressing as a single fraction and attempt to simplify to required form.

8(b)	**Alternative 2** Use of $\text{cosec}^2\,\theta = 1 + \cot^2\theta$ **and** $\cot^2\theta = \dfrac{1}{\tan^2\theta}$	**(B1)**	May be implied by later work
	$\tan^2\theta = \dfrac{4-\sqrt{3}}{7+4\sqrt{3}}$ $= 52 - 30\sqrt{3}$	**(2)**	**M1** for attempting to rationalise $\tan^2\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms
	$\text{cosec}^2\,\theta = 1 + \left(\dfrac{1}{52-30\sqrt{3}}\right)^2$ $= 14 + \dfrac{15\sqrt{3}}{2}$	**(2)**	**M1** for attempting to rationalise $\cot^2\theta$ with sufficient detail in the simplification of the numerator – at least 3 terms **and** expressing as a single fraction and attempt to simplify to required form.
	Alternative 3 Use of right-angled triangle $\text{Hyp}^2 = 11 + 2\sqrt{3}$	**(2)**	**M1** For attempt to calculate the square of hypotenuse
	$\text{cosec}^2\theta = \dfrac{11+2\sqrt{3}}{4-2\sqrt{3}}$	**(B1)**	for correct use of $\text{cosec}^2\,\theta$ with *their* squared hypotenuse
	$\text{cosec}^2\theta = \dfrac{11+2\sqrt{3}}{4-2\sqrt{3}} \times \dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$	**(M1)**	For attempt to rationalise *their* expression, with sufficient detail in the simplification of the numerator – at least 3 terms
	$14 + \dfrac{15\sqrt{3}}{2}$	**A1**	

60.

1	$a=5$	**B1**	
	$b=4$	**B1**	
	$c=-3$	**B1**	

61.

2	$\tan^2\theta = \dfrac{1}{y+2}$ soi **or** $x = 1+\tan^2\theta$ soi	**B1**	Must be in terms of $\tan^2\theta$
	Use of $\tan^2\theta + 1 = \sec^2\theta$ $\dfrac{1}{y+2} + 1 = x$ oe	**M1**	For a valid attempt to eliminate θ
	$y = \dfrac{1}{x-1} - 2$ or $y = \dfrac{3-2x}{x-1}$ oe	**2**	**Dep M1** for attempt to rearrange to obtain in the required form **A1** for a correct form
	Alternative $x = \dfrac{1}{\cos^2\theta}$ **and** $y+2 = \dfrac{\cos^2\theta}{\sin^2\theta}$ soi	**(B1)**	
	$y+2 = \dfrac{\frac{1}{x}}{1-\frac{1}{x}}$ oe	**(M1)**	For a valid attempt to eliminate θ, making use of $\sin^2\theta + \cos^2\theta = 1$
	$y = \dfrac{1}{x-1} - 2$ or $y = \dfrac{3-2x}{x-1}$ oe	**(2)**	**Dep M1** for attempt to rearrange to obtain in the required form **A1** for a correct form

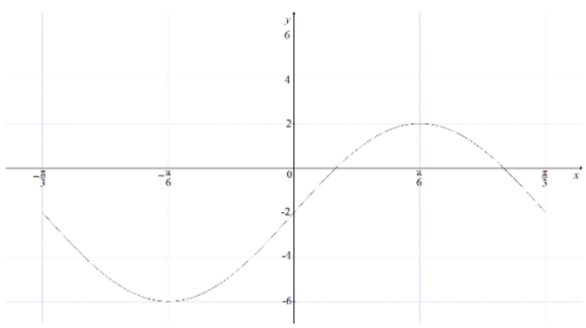
62.

1	$a=2$	**B1**	
	$b=3$	**B1**	
	$c=-4$	**B1**	

63.

4	$\tan\left(2x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ or $\sin^2\left(2x+\frac{\pi}{4}\right)=\frac{1}{4}$ or $\cos^2\left(2x+\frac{\pi}{4}\right)=\frac{3}{4}$	**B1**	Must be from correct working Allow if $\theta=2x+\frac{\pi}{4}$ oe
	$2x+\frac{\pi}{4}=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}$ $x=-\frac{\pi}{24}$	**M1**	**Dep** on previous **B1** For attempt at the correct order of operations, may be implied by a correct solution or $x=-\frac{\pi}{24}$.
	$x=\frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	2	**Dep M1** for an attempt to find a solution within the given range. Must be working with $\frac{7\pi}{6}$ or $\frac{13\pi}{6}$ **A1** for either
	$x=\frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	**A1**	For a second solution within the given range with no extra solutions within the range.

64.

1		**3**	**B1** for a curve starting at $\left(-\frac{\pi}{3},-2\right)$ and finishing at $\left(\frac{\pi}{3},-2\right)$ **B1** for a curve, must have implied symmetry about $\frac{\pi}{6}$ and $-\frac{\pi}{6}$, one complete cycle only. **B1** for a curve passing through (0, –2) and distinct maximum at $\left(\frac{\pi}{6},2\right)$ and distinct minimum at only $\left(-\frac{\pi}{6},-6\right)$

65.

10	$\tan(3x + 1.2)\left[=\frac{1}{\sqrt{2}}\right]$ or $\cos^2(3x + 1.2)\left[=\frac{2}{3}\right]$ or $\sin^2(3x + 1.2)\left[=\frac{1}{3}\right]$	**M1**	For an attempt to obtain an equation in $\sin(3x + 1.2)$, $\cos(3x + 1.2)$ or $\tan(3x + 1.2)$
	$x = -1.24, -0.195, 0.852$ or better	**4**	**M1 dep** for a correct attempt to obtain one correct solution **A1** for one correct solution in the range **M1 dep** for an attempt to obtain another solution within the range **A1** for 2 more correct solutions within the range and no extra solutions within the range

66.

7(a)	$\frac{\sin^2 x + (1-\cos x)^2}{(1-\cos x)\sin x}$ or $\frac{\sin^2 x}{(1-\cos x)\sin x} + \frac{(1-\cos x)^2}{(1-\cos x)\sin x}$	**M1**	
	$\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{(1-\cos x)\sin x}$	**A1**	OR $\frac{1-\cos^2 x + (1-\cos x)^2}{(1-\cos x)\sin x}$
	$\frac{1+1-2\cos x}{(1-\cos x)\sin x}$ or $\frac{1-\cos^2 x + 1 - 2\cos x + \cos^2 x}{(1-\cos x)\sin x}$	**A1**	OR $\frac{(1-\cos x)(1+\cos x) + (1-\cos x)^2}{(1-\cos x)\sin x}$
	Fully correct justification of given answer: $\frac{2(1-\cos x)}{(1-\cos x)\sin x} = 2\operatorname{cosec} x$ or $\frac{2-2\cos x}{(1-\cos x)\sin x} = \frac{2}{\sin x} = 2\operatorname{cosec} x$ or equivalent	**A1**	All steps correct and final step justified OR $\frac{1+\cos x + 1 - \cos x}{\sin x} = 2\operatorname{cosec} x$

	Alternative		
	$\frac{\sin x(1+\cos x)}{(1-\cos x)(1+\cos x)}+\frac{(1-\cos x)\sin x}{\sin x\sin x}$ or $\frac{\sin x(1+\cos x)}{1-\cos^2 x}+\frac{(1-\cos x)\sin x}{\sin^2 x}$	**(M1)**	
	$\frac{\sin x+\sin x\cos x}{\sin^2 x}+\frac{\sin x-\cos x\sin x}{\sin^2 x}$	**(A1)**	
	$\frac{2\sin x}{\sin^2 x}$	**(A1)**	
	Fully correct justification of given answer: $\frac{2}{\sin x}=2\operatorname{cosec} x$	**(A1)**	All steps correct and final step justified

Question	Answer	Marks	Guidance
7(b)	$3\sin^2 x-\sin x-2\quad[=0]$ soi	**B1**	
	$(3\sin x + 2)(\sin x - 1)$ [= 0] oe	**M1**	
	$\sin x=-\frac{2}{3}$, $\sin x=1$	**A1**	
	90 221.8 or 221.81[03…] rot to 2 or more dp 318.2 or 318.18[96…] rot to 2 or more dp	**A1**	and no extras in range If **B1 M1 A0 A0** allow **SC1** for 221.8 or 221.81[03…] rot to 2 or more dp **and** 318.2 or 318.18[96…] rot to 2 or more dp and no extras in range

67.

5(a)	$\dfrac{\cos^2 x+(1-\sin x)^2}{(1-\sin x)\cos x}$ or $\dfrac{\cos^2 x}{(1-\sin x)\cos x}+\dfrac{(1-\sin x)^2}{(1-\sin x)\cos x}$	M1	Correctly takes common denominator
	$\dfrac{\cos^2 x+1-2\sin x+\sin^2 x}{(1-\sin x)\cos x}$	A1	OR $\dfrac{1-\sin^2 x+(1-\sin x)^2}{(1-\sin x)\cos x}$
	$\dfrac{1+1-2\sin x}{(1-\sin x)\cos x}$ or $\dfrac{1-\sin^2 x+1-2\sin x+\sin^2 x}{(1-\sin x)\cos x}$	A1	OR $\dfrac{(1-\sin x)(1+\sin x)+(1-\sin x)^2}{(1-\sin x)\cos x}$
	$\dfrac{2(1-\sin x)}{(1-\sin x)\cos x}=2\sec x$ or $\dfrac{2-2\sin x}{(1-\sin x)\cos x}=\dfrac{2}{\cos x}=2\sec x$ or equivalent	A1	All steps correct and final step justified OR $\dfrac{1+\sin x+1-\sin x}{\cos x}=2\sec x$
	Alternative ***Must work with LHS only***		
	$\dfrac{(\cos x)(1+\sin x)}{(1-\sin x)(1+\sin x)}+\dfrac{(1-\sin x)\cos x}{(\cos x)\cos x}$	(M1)	Forms fractions with common denominator in different form
	$\dfrac{(\cos x)(1+\sin x)}{\cos^2 x}+\dfrac{(1-\sin x)\cos x}{\cos^2 x}$	(A1)	Uses difference of two squares and $\sin^2 x+\cos^2 x=1$ to write fractions with a common denominator in the same form
	$\dfrac{2\cos x}{\cos^2 x}$	(A1)	Combine as a single fraction and collects terms
	$\dfrac{2}{\cos x}=2\sec x$	(A1)	All steps correct and final step justified

5(b)	$\cos^3\frac{\theta}{2}=\frac{1}{4}$	B1	
	$\cos\frac{\theta}{2}=\sqrt[3]{their\frac{1}{4}}$ soi	M1	**dep** on starting with $2\sec\frac{\theta}{2}=8\cos^2\frac{\theta}{2}$
	±101.9 awrt	A2	and no extras in range **A1** for either, ignoring extras in range If **A0** then **SC1** for ±102 with no extras in range

68.

6(a)	$\dfrac{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$ soi	2	**B1** for tan θ and cot θ in terms of sin and cos. **B1** for $\sec\theta=\frac{1}{\cos\theta}$
	$\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}\times\cos\theta$ soi oe	M1	For dealing with the fractions in the numerator.
	$\frac{1}{\sin\theta}\times$ cosec θ cso	A1	For correct use of $\cos^2\theta+\sin^2\theta=1$ to obtain the given answer.
	$\dfrac{\frac{1}{\tan\theta}+\tan\theta}{\frac{1}{\cos\theta}}=\frac{1+\tan^2\theta}{\tan\theta}\times\cos\theta$ soi oe	(2)	**B1** for sec $\theta=\frac{1}{\cos\theta}$ **M1** for dealing with the fractions in the numerator.
	$\frac{\sec^2\theta}{\tan\theta}\times\cos\theta$	(B1)	For correct use of $\tan^2\theta+1=\sec^2\theta$
	$\frac{1}{\sin\theta}\times$ cosec θ cso	(A1)	For correct use of tan θ and $\sec^2\theta$ to obtain the given answer.

6(b)	$\left(\dfrac{1}{\sin\frac{\phi}{3}}\right)^2 = 2$ or $\sin\frac{\phi}{3} = \pm\frac{1}{\sqrt{2}}$ soi or $\tan\frac{\phi}{3} = \pm 1$ soi	**B2**	**B1** for ± missing
	–405°, –135°, 135°, 405°	4	**M1** for one correct positive or negative solution of *their* $\sin\frac{\phi}{3} = k$ **A1** for another correct solution **M1Dep** for one negative or positive solution **A1** for another correct solution and no extras in the range.

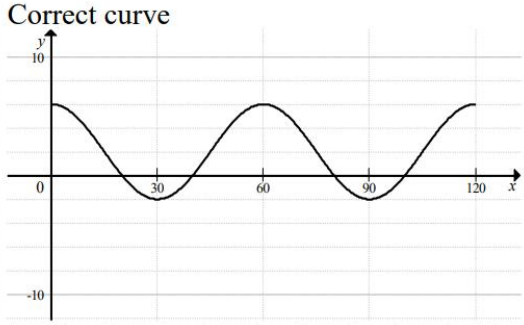
69.

1	$a = 4$	**B1**	
	$b = \frac{3}{8}$ oe	**B1**	
	$c = -2$	**B1**	

70.

6(a)	A correct equation in terms of *x* and *y* only	**B1**	No inverse trig functions
	$y = (x-4)^2 - 3$ or $y = x^2 - 8x + 13$	**B1**	
6(b)	$\sin\left(2\phi + \frac{3\pi}{4}\right) = \frac{\sqrt{3}}{2}$ soi	**B1**	May be implied by one correct solution
	$-\frac{5\pi}{24}, -\frac{\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}$ with no extra solutions within the range	4	**M1** for explicitly correct order of operations from *their* $\left(2\phi + \frac{3\pi}{4}\right) = k$, or may be implied by one correct solution **A1** for two correct solutions **A1** for a third correct solution **A1** for a further solution with no extra solutions in the range

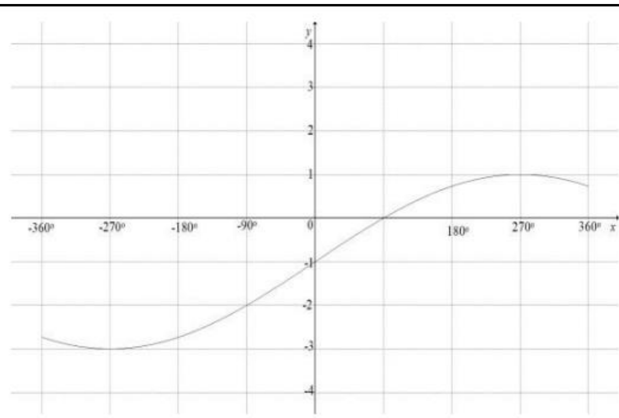
71.

2(a)	Correct curve 	3	**B2** for correct cosine shape over 2 cycles with midline at $y = 2$ and consistent amplitude or **B1** for attempt at cosine shape over 2 cycles **with** consistent amplitude **B1** for a consistent amplitude of 2; must have attempted correct shape **Maximum of 2 marks if not fully correct**
2(b)	4	1	
2(c)	60°	1	

72.

5	$\cos\left(2\theta+\frac{\pi}{6}\right)=(\pm)\frac{\sqrt{3}}{2}$ oe or $\tan\left(2\theta+\frac{\pi}{6}\right)=(\pm)\frac{1}{\sqrt{3}}$ oe	**B1**	
	$\theta = -\frac{\pi}{6},\ 0,\ \frac{\pi}{3}$ oe	4	**M1** for a correct order of operations, may be implied by one correct solution. **A1** for 1 correct solution. **A1** for a 2nd correct solution **A1** for a 3rd correct solution with no extra solutions in the range. All solutions must be from correct working.

73.

3		4	**B1** for a correct shape, starting in approximately correct places between -2 and -3 and finishing in approximately correct places between 0 and 1, having an amplitude of 2 and crossing the x-axis only once, on the positive x-axis. **B1** for a correct shape and $(0,\ -1)$ **B1** for a correct shape and a max and a min in approximately correct places. $(270^\circ, 1)$ and $(-270^\circ, -3)$ **B1** for a correct shape crosses at $(90^\circ, 0)$

74.

12	$\sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\frac{\sqrt{3}}{2}$ or $\tan\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\sqrt{3}$	**B1**	Allow if $\pm$ is missing
	$x=\pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$	**4**	**M1dep** on **B1** for obtaining $\frac{2x}{3}-\frac{\pi}{3}=\frac{\pi}{3}$ or any valid value **A1** for one correct solution **A1** for a 2nd correct solution **A1** for a 3rd and 4th correct solutions and no extras in the range

75.

Question	Answer	Marks	Guidance
10(a)	Writes cot$x$ and tan$x$ in terms of sin$x$ and cos$x$: $\frac{\sin x}{1-\frac{\cos x}{\sin x}}+\frac{\cos x}{1-\frac{\sin x}{\cos x}}$	**M1**	OR $\frac{\sin x\left(1-\frac{\sin x}{\cos x}\right)+\cos x\left(1-\frac{\cos x}{\sin x}\right)}{\left(1-\frac{\cos x}{\sin x}\right)\left(1-\frac{\sin x}{\cos x}\right)}$
	Simplifies denominator: $\frac{\sin x}{\frac{\sin x-\cos x}{\sin x}}+\frac{\cos x}{\frac{\cos x-\sin x}{\cos x}}$	**A1**	OR $\frac{\sin x\left(\frac{\cos x-\sin x}{\cos x}\right)+\cos x\left(\frac{\sin x-\cos x}{\sin x}\right)}{\left(\frac{\sin x-\cos x}{\sin x}\right)\left(\frac{\cos x-\sin x}{\cos x}\right)}$
	Writes as two simple algebraic fractions: $\frac{\sin^2 x}{\sin x-\cos x}+\frac{\cos^2 x}{\cos x-\sin x}$	**A1**	OR writes as a single simple algebraic fraction: $\frac{\sin^2 x(\cos x-\sin x)+\cos^2 x(\sin x-\cos x)}{(\sin x-\cos x)(\cos x-\sin x)}$
	Writes as a difference with a common denominator: $\frac{\sin^2 x}{\sin x-\cos x}-\frac{\cos^2 x}{\sin x-\cos x}$	**A1**	OR $\frac{\sin^2 x(\cos x-\sin x)-\cos^2 x(\cos x-\sin x)}{(\sin x-\cos x)(\cos x-\sin x)}$
	Correct simplification to given answer, e.g., $\frac{(\sin x-\cos x)(\sin x+\cos x)}{(\sin x-\cos x)}=\sin x+\cos x$ or $\frac{\cancel{(\sin x-\cos x)}(\sin x+\cos x)}{\cancel{(\sin x-\cos x)}}[=\sin x+\cos x]$	**A1**	All steps correct and final step fully justified by factorising

Question	Answer	Marks	Guidance
10(b)	$10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$	**B2**	**B1** for $\frac{9\cos x}{\sin x} + \frac{3}{\sin x} = \frac{\sin x}{\cos x}$ or better or $9 + \frac{3\tan x}{\sin x} = \tan^2 x$ or better OR **M1** for one sign error in $10\cos^2 x + 3\cos x - 1 [= 0]$ or $\sec^2 x - 3\sec x - 10 [= 0]$
	$(5\cos x - 1)(2\cos x + 1)[= 0]$ or $(\sec x - 5)(\sec x + 2)[= 0]$	**M1**	**FT** *their* 3-term quadratic in cos$x$ or sec$x$
	[cos$x$ = $\frac{1}{5}$ **and** cos$x$ = $-\frac{1}{2}$ OR sec$x$ = 5 **and** sec$x$ = –2 leading to] 78.5 or 78.46[30…] rot to 2 or more dp 281.5 or 281.53[69…] rot to 2 or more dp 120 240 and no extras in range $0 < x < 360$	**A2**	**A1** for any two correct angles [found using cos$x$ = $\frac{1}{5}$ **and** cos$x$ = $-\frac{1}{2}$ OR sec$x$ = 5 **and** sec$x$ = –2]; ignore extras