1.

5(i)	$^{10}C_4 = 210$	**B1**	
5(ii)	2 Mystery 2 others = $^5C_2 \times {}^5C_2$ =100 3 Mystery 1 other = $^5C_3 \times {}^5C_1$ = 50 4 Mystery = $^5C_4$ =5 Total 155	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total
	Alternative Method All – 0 Mystery – 1 Mystery	**B1**	All minus 0 or 1 or both
	$= 210 - {}^5C_4 - {}^5C_1 \times {}^5C_3$	**B1**	**B1dep** 1Mystery and 0 mystery unsimplified
	$= 210 - 5 - 5 \times 10 = 155$	**B1**	**B1dep** final answer
5(iii)	2M1C1R = $^5C_2 \times {}^3C_1 \times {}^2C_1 = 60$ 1M2C1R = $^5C_1 \times {}^3C_2 \times {}^2C_1 = 30$ 1M1C2R $= {}^5C_1 \times {}^3C_1 \times {}^2C_2$ $= 15$ Total 105	**B3**	**B1** for one combination, unsimplifiied **B1** for second combination, unsimplifiied **B1** for third combination, unsimplifiied and total

2.

9(a)(i)	720	**B1**	
9(a)(ii)	240	**B1**	
9(a)(iii)	$k \times 4! \times 2$ or $240 - k \times 4! \times 2$ or correct equivalents with no extra terms added or subtracted	**B1**	
	$4 \times 4! \times p$ or correct equivalents with no extra terms added or subtracted	**B1**	
	192	**B1**	
9(b)(i)	6435	**B1**	
9(b)(ii)	With twins: $^{13}C_6$ or 1716 Without twins: $^{13}C_8$ or 1287	**B2**	**B1** for $^{13}C_6$ or 1716 or $^{13}C_8$ or 1287 **B1** for ($^{13}C_6$ and $^{13}C_8$) or (1716 and 1287) with no multiples and no extra terms
	Total: 1716 + 1287 = 3003	**B1**	3003 from a correct method

3.

8(a)	$\left[{}^{30}P_2 =\right]$ 870	**B1**	
8(b)(i)	${}^2C_1 \times {}^{14}C_{10}$ oe (2 × 1001)	**M1**	Condone $\begin{pmatrix}14\\4\end{pmatrix}$ for $\begin{pmatrix}14\\10\end{pmatrix}$
	2002	**A1**	implies **M1**
8(b)(ii)	$\left({}^2C_1 \times {}^5C_4 \times {}^9C_6\right) + \left({}^2C_1 \times {}^5C_5 \times {}^9C_5\right)$ oe (840 + 252) or ${}^2C_1 \times {}^{14}C_{10} -$ $\left({}^2C_1 \times {}^5C_1 \times {}^9C_9 + {}^2C_1 \times {}^5C_2 \times {}^9C_8 + {}^2C_1 \times {}^5C_3 \times {}^9C_7\right)$ {2002 – (10 + 80 + 720)}	**M3**	**M3 f**or fully correct method soi **M2 f**or all necessary products but **n**ot summed with no extra products seen soi **M1 f**or one correct three term product soi
	1092	**A1**	implies **M3**

4.

8(a)(i)	2520	**B1**	
8(a)(ii)	360	**B1**	
8(a)(iii)	1080	**B1**	
8(a)(iv)	6 or 8 to start with No of ways = $2 \times 5 \times 4 \times 3 \times 2$ = 240	**B1**	
	9 to start with No of ways = $1 \times 5 \times 4 \times 3 \times 3$ = 180	**B1**	
	Total number of ways = 420	**DB1**	Dependent on both previous B marks

8(a)(iv)	**Alternative 1** All numbers > 6000 – all odd numbers > 6000	**B1**	plan and attempt to use, must be using 1080
	$1080 - 180 - 480$	**B1**	for 180 and 480
	Total number of ways = 420	**DB1**	Dependent on both previous B marks
	Alternative 2 Even numbers > 60000 : Odd numbers > 60000 7 : 11	**B1**	correct ratio
	Total number of ways = $\frac{7}{18} \times 1080$	**B1**	
	= 420	**DB1**	Dependent on both previous B marks
8(b)(i)	480700	**B1**	
8(b)(ii)	26460	**B1**	
8(b)(iii)	With brother and sister $^{23}C_5 = 33649$	**B1**	for $^{23}C_5$ or $^{23}C_5 \times {}^kC_k$
	Without brother and sister $^{23}C_7 = 245157$	**B1**	for $^{23}C_7$ or $^{23}C_7 \times {}^kC_k$
	Total number of ways = 278806	**B1**	for $^{23}C_5 + {}^{23}C_7$ and evaluation

5.

7(i)	167 960	**1**	
7(ii)	evidence of selecting from 16	**M1**	
	[$^{16}C_7$ =] 11 440	**A1**	
7(iii)	$2 \times {}^nC_r$ with $n = 16$ or $r = 9$	**M1**	
	$\left[2 \times {}^{16}C_9 =\right]$ 22880	**A1**	
7(iv)	$4 \times {}^nC_r$ with $n = 16$ or $r = 9$	**M1**	
	$\left[4 \times {}^{16}C_9 =\right]$ 45760	**A1**	

6.

6(a)	Number first $=7\times6\times5\times6\times5$ or ${}^7P_3\times{}^6P_2$ or 6300	**B1**	
	Letter first $=6\times5\times4\times7\times6$ or ${}^6P_3\times{}^7P_2$ or 5040	**B1**	
	$6300+5040=11\,340$	**B1**	
6(b)	With 2 sisters = ${}^7C_5\times{}^3C_2=63$ With 1 sister = ${}^7C_6\times{}^3C_1=21$ With no sister = ${}^7C_7=1$ and Total 85	**3**	**B1** One combination evaluated **B1**Another combination evaluated **B1** Third combination and 85
	OR		
	Total no of ways = ${}^{10}C_7=120$	**B1**	
	With 3 sisters = ${}^7C_4=35$	**B1**	
	Without 3 sisters = $120-35=85$	**B1**	

7.

5(a)	${}^7P_4$ or $7\times6\times5\times4$ oe	**M1**	
	840	**A1**	
5(b)(i)	20	**B1**	
5(b)(ii)	${}^5C_1\times{}^4C_1\times{}^2C_1$ or $5\times4\times2$ oe	**M1**	
	40	**A1**	
5(b)(iii)	${}^5C_3+{}^4C_3$ oe	**M1**	
	14	**A1**	

8.

3(i)	$({}^{12}P_7=)\,3\,991\,680$	**B1**	
3(ii)	$(4\times{}^{11}P_6=)\,1\,330\,560$	**B1**	
3(iii)	$4!\times4!\times2$ oe	**M2**	**M1** for $4!\times4!$ oe only or ${}^4P_4\times{}^4P_3$ oe only
	1152	**A1**	

9.

3(i)	${}^{14}P_5$ or $14 \times 13 \times 12 \times 11 \times 10$	**M1**	
	240 240	**A1**	cao
3(ii)	${}^{3}P_1 \times {}^{5}P_2 \times {}^{6}P_2$ or $3 \times (5 \times 4) \times (6 \times 5)$	**M1**	Two of the three elements multiplied by …
	= 1800	**A1**	
3(iii)	${}^{6}P_2 \times {}^{8}P_3$ or $(6 \times 5) \times (8 \times 7 \times 6)$	**M1**	One element multiplied by … Clear intention to multiply
	= 10 080	**A1**	

10.

7(a)(i)	15 120	**B1**	
7(a)(ii)	1680	**B1**	
7(a)(iii) Method 1	Total = 2310	**B3**	B1 1st digit is 7 or 9 1680 or 210×8 B1 1st digit is 8 630 or 210×3
7(a)(iii) Method 2	Total = 2310	**B3**	B1 for 5th digit is 2,4 or 6 1890 or 210×9 B1 for 5th digit is 8 420 or 210×2

Question	Answer	Marks	Guidance
7(b)(i)	3003	**B1**	
7(b)(ii)	28	**B1**	
7(b)(iii)	Total 1419	**B3**	B1 Including husband and wife 495 B1 Excluding husband and wife 924

11. OMITTED

12.

9(a)(i)	39 916 800	**B1**	
9(a)(ii)	$5! \times 6!$ oe	**M1**	
	86 400	**A1**	
9(b)(i)	${}^{5}C_3 \times {}^{3}C_1$ oe	**M1**	
	30	**A1**	

13.

6(a)(i)	40320	**B1**	
6(a)(ii)	No. of ways with maths books as 1 unit = $5!$ or $5\times4!$ or ${}^5P_5$ or 120	**B1**	
	No. of ways maths books can be arranged amongst themselves = $4!$ or ${}^4P_4$ or 24	**B1**	
	Total = $(5!\ \times4!\ \text{oe})=2880$	**B1**	
6(a)(iii)	No. of ways with maths books as 1 unit and geography books as 1 unit = $3!$ or ${}^3P_3$ or $3\times2!$ or 6	**B1**	
	No. of ways maths books can be arranged amongst themselves and geography books can be arranged amongst themselves $=4!\ \times3!$ or ${}^4P_4\times{}^3P_3$ or 144	**B1**	
	Total = $(3!\ \times4!\ \times3!\ \text{oe})$ = 864	**B1**	
6(b)(i)	${}^{12}C_6=924$	**B1**	

Question	Answer	Marks	Guidance
6(b)(ii)	**Either:** $924-{}^8C_6$	**M1**	For *their* **(i)** – the number of teams of just men
	Total = 896	**A1**	
	Or: 5M 1W : ${}^8C_5\times{}^4C_1$ $(=224)$ 4M 2W : ${}^8C_4\times{}^4C_2$ $(=420)$ 3M 3W : ${}^8C_3\times{}^4C_3$ $(=224)$ 2M 4W : ${}^8C_2\times{}^4C_4$ $(=28)$	**M1**	For a complete method
	Total = 896	**A1**	

14.

6(a)	3024	**B1**	
6(b)	24	**B1**	
6(c)	${}^{4}P_{2} \times {}^{5}P_{2}$	**M1**	$4\times3 \times 5\times4$
	240 no isw	**A1**	
6(d)	${}^{4}P_{1} \times {}^{8}P_{3}$	**M1**	$4 \times 8\times7\times6$
	1344 no isw	**A1**	

15.

8(b)(i)	120	**B1**	
8(b)(ii)	48	**B1**	
8(b)(iii)	Starts with 7 or 9 24	**B1**	May be implied by 12 and 12
	Starts with 8 18	**B1**	
	42	**B1**	
	Alternative Ends with 3 18	**(B1)**	
	Ends with 7 or 9 24	**(B1)**	May be implied by 12 and 12
	42	**(B1)**	

16.

4(a)	${}^{5}C_{2}\times{}^{8}C_{4}$ oe	**M1**	
	700	**A1**	
4(b)	$3\times6!$ oe	**M1**	
	2160	**A1**	

17.

4(a)(i)	720	**B1**	
4(a)(ii)	600	**B1**	**FT** on *their* (i) $\times\frac{5}{6}$
4(a)(iii)	Starting with 8: $1\times4\times3\times2\times1=24$	**B1**	
	Starting with 3, 5 or 7: $3\times4\times3\times2\times2=144$	**M1**	May be considering each case separately, need all three cases for M1
		A1	
	$\text{Total}=168$	**A1**	
4(a)(iii)	**Alternative** Plan for adding numbers ending in 2 and numbers ending in 8	**M1**	
	Ending in 2: $\left(\frac{1}{6}\times720\right)\times\frac{4}{5}=96$	**B1**	Allow unsimplified
	Ending in 8: $\left(\frac{1}{6}\times720\right)\times\frac{3}{5}=72$	**B1**	Allow unsimplified
	$\text{Total}=168$	**A1**	
4(b)	${}^nC_3=6\,{}^nC_2$	**B1**	$\frac{n(n-1)(n-2)}{3!}$
	$\frac{n(n-1)(n-2)}{3!}=\frac{6n(n-1)}{2!}$	**B1**	$\frac{6n(n-1)}{2!}$
	$n(n-1)\left[(n-2)-18\right]=0$	**M1**	Valid attempt to solve, must have at least one previous B mark
	$n=20$	**A1**	
4(b)	**Alternative** ${}^nC_3=6\,{}^nC_2$	**B1**	For dealing with $(n-2)!$ and $(n-3)!$

18.

Question	Answer	Marks	Guidance
6(a)	**Either** Starts with 8: 1680	**B1**	1680 must not be part of a product. May be implied by final answer
	Starts with 7 or 9: 2688	**B1**	May be implied by final answer
	Total: 4368	**B1**	
	Or Alternative 1 Starts with 7, 8 or 9 and ends in 1, 3 or 5: 3024	**(B1)**	Allow for 1008 three times May be implied by final answer
	Starts with 8 or 9 and ends in 7: 672 Starts with 7 or 8 and ends in 9: 672	**(B1)**	For both May be implied by final answer
	Total: 4368	**(B1)**	
	Or Alternative 2 13 ways of obtaining odd 5-digit numbers which start with 7, 8 or 9	**(B1)**	Needs to be part of a product. May be implied by final answer
	${}^{8}P_{3}$ ways of arranging the remaining 3 digits: 336	**(B1)**	Needs to be part of a product. May be implied by final answer
	Total = $13 \times 336 = 4368$	**(B1)**	
	Or Alternative 3 Last digit is 7 or 9: 1344	**B1**	May be implied by final answer
	Last digit is 1, 3 or 5: 3024	**B1**	May be implied by final answer
	Total: 4368	**B1**	
	Or Alternative 4 ${}^{10}P_{5} - ({}^{9}P_{4} \times 7) - ({}^{8}P_{3} \times 5) - ({}^{8}P_{3} \times 4)$ $-({}^{8}P_{3} \times 5)$	**B2**	Must be complete
	Total: 4368	**B1**	

6(b)	$\frac{n!}{(n-3)!3!}=\frac{2n!}{(n-2)!2!}$ soi	**B1**	
	$(n-2)=6$ soi	**B2**	**Dep B1** on first **B** for $(n-2)$ soi **Dep B1** on first **B** for 6 soi
	$n=8$	**B1**	**Dep** on previous **B** marks

19.

8(a)	**Either** Starting with a 6: 120 ways	**B1**	May be implied by final answer
	Starting with 5, 7 or 9: 540 ways	**B1**	May be implied by final answer
	Total 660	**B1**	
	Or Alternative 1 Ending with a 6: 180 ways	**(B1)**	May be implied by final answer
	Ending with 0 or 4: 480ways	**(B1)**	May be implied by final answer
	Total 660	**(B1)**	
	Or Alternative 2 11 ways of obtaining even 5-digit numbers which start with 5, 6, 7, 9	**(B1)**	For $11 \times k$ May be implied by final answer
	${}^{5}P_{3}$ ways of arranging remaining 3 digits: 60	**(B1)**	For $m \times 60$ where m is from an attempt to list all cases for first and last digits May be implied by final answer
	$11 \times 60 = 660$	**(B1)**	
	Or Alternative 3 Total arrangements ${}^{7}P_{5}$ minus (all odds + evens starting with 1 + evens starting with 0 or 4) = 2520 – (1440 + 180 + 240)	**(B2)**	For 2520 – (1440 + 180 + 240)
	660	**(B1)**	

Question	Answer	Marks	Guidance
8(b)	$\frac{n!}{(n-4)!4!}=\frac{6n!}{(n-2)!2!}$	**B1**	
	$(n-2)(n-3)=72$	**2**	**B1** for $(n-2)(n-3)$

20.

5(a)(i)	120	**B1**	
5(a)(ii)	$4 \times 3 \times 2 \times 1 \times 3$ =72 odd numbers 60%	**2**	**B1** for either: 72 or $4 \times 3 \times 2 \times 1 \times 3$ and no percentage, or $\frac{3}{5}$ and no percentage.
5(b)	495	**B1**	
	$495 \times 70 \ (= 34650)$	**B1**	
	$\left(\frac{34650}{3!} = \right)$ 5775	**B1**	

21.

7(a)(i)	20160	**B1**	
7(a)(ii)	7200	**2**	**B1** for ${}^6P_4$ or $6 \times 5 \times 4 \times 3 (= 360)$ for 'inner' characters or ${}^5P_2$ or $4 \times 5 (= 20)$ for 'outer' characters Must be part of a product
7(a)(iii)	360	**2**	**B1** for ${}^3P_3$ or 3! or 6 for arrangements of symbols or ${}^5P_3$ or $5 \times 4 \times 3$ (= 60) for the digits Must be part of a product
7(b)	$\frac{n!}{(n-5)!5!} = \frac{6(n-1)!}{((n-1)-4)!4!}$	**B1**	May be implied by simplification e.g. $\frac{n!}{5!} = 6\frac{(n-1)!}{4!}$ or $\frac{n(n-1)(n-2)(n-3)(n-4)}{}$

22.

6(a)(i)	720	**B1**	
6(a)(ii)	480	**B1**	

Question	Answer	Marks	Guidance
6(a)(iii)	[Starts with 6 or 8]: 192	**B1**	
	[Starts with 9]: 72	**B1**	
	Total = 264	**B1**	
	Alternative [Ends with 9]:48	**(B1)**	
	[Ends with 1,3 or 5]:216	**(B1)**	
	Total = 264	**(B1)**	
6(b)	$\frac{45n!}{(n-4)!4!} = \frac{(n+1)(n+1)!}{((n+1)-5)!5!}$	**B1**	
	$45 = \frac{(n+1)^2}{5}$ leading to $15 = n+1$ or $n^2 + 2n - 224 = 0$	**M2**	**M1** for 15 **M1** for $n + 1$ OR **M1** for $n^2 + 2n - 224 = 0$ oe **M1** for $(n-14)(n+16) = 0$
	$n = 14$ only	**A1**	

23.

9(a)(i)	665 280	**B1**	
9(a)(ii)	221 760	**B1**	

Question	Answer	Marks	Guidance
9(b)	$8 \times 4 \times 3 \times 2 \times 1 \times 7$	**M1**	For either 8×7 or 4! or 24 as part of a product
	1344	**A1**	

24.

6	1144	**3**	**B1** With the brothers: 220 or ${}^{12}C_3$ **B1** Without the brothers: 924 or ${}^{12}C_6$

25.

Question	Answer	Marks	Guidance
6(a)	510	**3**	**M2** for a fully correct method e.g. [starts with 5, 7, 9 and ends in 3 or two of 5, 7, 9] $3 \times 6 \times 5 \times 3$ or 270 **and** [starts with 6, 8 and ends in 3,5,7, 9] $2 \times 6 \times 5 \times 4$ or 240 **OR** [ends with 3 and starts with 5, 6, 7, 8, 9] $5 \times 6 \times 5 \times 1$ or 150 **and** [ends with 5, 7, 9 and starts with 6, 8 or two of 5, 7, 9] $4 \times 6 \times 5 \times 3$ or 360 or **M1** for a partially correct method equivalent to one of the above two steps
6(b)	540	**3**	**M2** for a fully correct method e.g. [starts with 5, 7 and ends with 2, 3 and 5 or 7] $2 \times 6 \times 5 \times 3 = 180$ **and** [starts with 6, 8, 9 and ends with 2, 3,

26.

Question	Answer	Marks	Partial Marks
6(a)(i)	$6 \times 6 \times 5 \times 4 \times 3$ oe	**M1**	
	2160	**A1**	
6(a)(ii)	Full correct calculation (360 + 900) 'how many end with 0'+ 'how many end with 2, 4, 6' oe	**M2**	**M1** for any correct product soi Eg $(6 \times 5 \times 4 \times 3 \times 1)$ or 360 $(5 \times 5 \times 4 \times 3 \times 1)$ or 300 $(5 \times 5 \times 4 \times 3 \times 3)$ or 900
	1260 cao	**A1**	
6(b)	${}^{15}C_7 - {}^{9}C_7$	**M1**	
	6399	**A1**	
6(c)(i)	$\frac{n!}{(n-3)!3!} + \frac{n!}{(n-2)!2!}$	**B2**	**B1** for either expression correct
	$\frac{n(n-1)(n-2)}{6} + \frac{n(n-1)}{2}$	**M2**	**M1** for either expression correct
	$\frac{n(n-1)(n-2+3)}{6}$ or $\frac{n^3-3n^2+2n}{6} + \frac{3n^2-3n}{6}$ leading to $\frac{1}{6}(n^3-n)$	**A1**	
6(c)(ii)	$n(n^2-25)=0$ oe	**M1**	
	$n=5$ as the only solution	**A1**	

27.

8(a)	With the sisters: 70 or ${}^{8}C_4$ oe	B1	
	Without the sisters: 28 or ${}^{8}C_6$ oe	B1	
	Total: 98	B1	
8(b)(i)	60480	B1	
8(b)(ii)	The start of the password and the end of the password can each be chosen 6 ways	B1	6 or ${}^{3}P_2$ oe seen twice
	The remaining characters can be chosen in 20 ways	B1	20 or ${}^{5}P_2$ oe seen
	Total number of ways: 720	B1	

28.

7(a)	${}^{14}C_2 \times {}^{12}C_3 \times {}^{9}C_4$ oe, soi 2 522 520	3	**B1** for a product of 3 combinations (ignore combinations that are equal to 1), one of which must be in the form ${}^{14}C_k$ where $k = 2, 3, 4, 5, 9, 10, 11, 12$ **B2** for a correct product of combinations
7(b)(i)	136 080	B1	
7(b)(ii)	15 120	B1	
7(b)(iii)	38 640	3	**B1** for ${}^{8}P_4$ or 1680 or $(8\times7\times6\times5)$ **B2** for $8\times{}^{8}P_4$ (13 440) oe or $15\times{}^{8}P_4$ (25 200) oe

29.

7(a)(i)	6435	B1	Must be evaluated not just ${}^{15}C_8$
7(a)(ii)	With family of 4: 330	B1	Must be evaluated not just ${}^{11}C_4$ or implied by a correct answer
	Without family of 4: 165	B1	Must be evaluated not just ${}^{11}C_8$ or implied by a correct answer
	Total: 495	B1	
7(b)	$\frac{(n+9)\times n!}{(n-10)!} = \frac{(n^2+243)(n-1)!}{(n-1-9)!}$ $n(n+9) = n^2 + 243$ oe	2	**B1** for either ${}^{n}P_{10} = \frac{n!}{(n-10)!}$ *or* ${}^{n}P_{10} = \frac{(n-1)!}{(n-1-9)!}$