1 Naomi is going on holiday and intends to read 4 books during her time away. She selects these books from 5 mystery, 3 crime and 2 romance books. Find the number of ways in which she can make her selection in each of the following cases.

(i) There are no restrictions. [1]

(ii) She selects at least 2 mystery books. [3]

(iii) She selects at least 1 book of each type. [3]

2 **(a)** A 6-digit number is to be formed using the digits 1, 3, 5, 6, 8, 9. Each of these digits may be used only once in any 6-digit number. Find how many different 6-digit numbers can be formed if

(i) there are no restrictions, [1]

(ii) the number formed is even, [1]

(iii) the number formed is even and greater than 300 000. [3]

(b) Ruby wants to have a party for her friends. She can only invite 8 of her 15 friends.

(i) Find the number of different ways she can choose her friends for the party if there are no restrictions. [1]

Two of her 15 friends are twins who cannot be separated.

(ii) Find the number of different ways she can now choose her friends for the party. [3]

3 **(a)** A football club has 30 players. In how many different ways can a captain and a vice-captain be selected at random from these players? [1]

(b) A team of 11 teachers is to be chosen from 2 mathematics teachers, 5 computing teachers and 9 science teachers. Find the number of different teams that can be chosen if

(i) the team must have exactly 1 mathematics teacher, [2]

(ii) the team must have exactly 1 mathematics teacher and at least 4 computing teachers. [4]

4 **(a)** A 5-digit number is to be formed from the seven digits 1, 2, 3, 5, 6, 8 and 9. Each digit can only be used once in any 5-digit number. Find the number of different 5-digit numbers that can be formed if

(i) there are no restrictions, [1]

(ii) the number is divisible by 5, [1]

(iii) the number is greater than 60 000, [1]

(iv) the number is greater than 60 000 and even. [3]

(b) Ranjit has 25 friends of whom 15 are boys and 10 are girls. Ranjit wishes to hold a birthday party but can only invite 7 friends. Find the number of different ways these 7 friends can be selected if

(i) there are no restrictions, [1]

(ii) only 2 of the 7 friends are boys, [1]

(iii) the 25 friends include a boy and his sister who cannot be separated. [3]

5 A squad of 20 boys, which includes 2 sets of twins, is available for selection for a cricket team of 11 players. Calculate the number of different teams that can be selected if

(i) there are no restrictions, [1]

(ii) both sets of twins are selected, [2]

(iii) one set of twins is selected but neither twin from the other set is selected, [2]

(iv) exactly one twin from each set of twins is selected. [2]

6 **(a)** A 5-character code is to be formed from the 13 characters shown below. Each character may be used once only in any code.

Letters : A, B, C, D, E, F

Numbers: 1, 2, 3, 4, 5, 6, 7

Find the number of different codes in which no two letters follow each other and no two numbers follow each other. [3]

(b) A netball team of 7 players is to be chosen from 10 girls. 3 of these 10 girls are sisters. Find the number of different ways the team can be chosen if the team does not contain all 3 sisters. [3]

7 **(a)** Four parts in a play are to be given to four of the girls chosen from the seven girls in a drama class. Find the number of different ways in which this can be done. [2]

(b) Three singers are chosen at random from a group of 5 Chinese, 4 Indian and 2 British singers. Find the number of different ways in which this can be done if

(i) no Chinese singer is chosen, [1]

(ii) one singer of each nationality is chosen, [2]

(iii) the three singers chosen are all of the same nationality. [2]

8 A 7-character password is to be selected from the 12 characters shown in the table. Each character may be used only once.

	Characters			
Upper-case letters	A	B	C	D
Lower-case letters	e	f	g	h
Digits	1	2	3	4

Find the number of different passwords

(i) if there are no restrictions, [1]

(ii) that start with a digit, [1]

(iii) that contain 4 upper-case letters and 3 lower-case letters such that all the upper-case letters are together and all the lower-case letters are together. [3]

9 A 5-digit code is formed using the following characters.

Letters	a	e	i	o	u	
Numbers	1	2	3	4	5	6
Symbols	@	*	#			

No character can be repeated in a code. Find the number of possible codes if

(i) there are no restrictions, [2]

(ii) the code starts with a symbol followed by two letters and then two numbers, [2]

(iii) the first two characters are numbers, and no other numbers appear in the code. [2]

10 **(a)** A 5-digit code is to be chosen from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit may be used once only in any 5-digit code. Find the number of different 5-digit codes that may be chosen if

(i) there are no restrictions, [1]

(ii) the code is divisible by 5, [1]

(iii) the code is even and greater than 70 000. [3]

(b) A team of 6 people is to be chosen from 8 men and 6 women. Find the number of different teams that may be chosen if

(i) there are no restrictions, [1]

(ii) there are no women in the team, [1]

(iii) there are a husband and wife who must not be separated. [3]

11 **(a)** Five teams took part in a competition in which each team played each of the other 4 teams. The following table represents the results after all the matches had been played.

Team	Won	Drawn	Lost
A	2	1	1
B	1	3	0
C	1	1	2
D	0	1	3
E	3	0	1

Points in the competition were awarded to the teams as follows

4 for each match won, 2 for each match drawn, 0 for each match lost.

(i) Write down two matrices whose product under matrix multiplication will give the total number of points awarded to each team. [2]

(ii) Evaluate the matrix product from **part (i)** and hence state which team was awarded the most points. [2]

12 **(a)** Eleven different television sets are to be displayed in a line in a large shop.

(i) Find the number of different ways the televisions can be arranged. [1]

Of these television sets, 6 are made by company *A* and 5 are made by company *B*.

(ii) Find the number of different ways the televisions can be arranged so that no two sets made by company *A* are next to each other. [2]

(b) A group of people is to be selected from 5 women and 3 men.

(i) Calculate the number of different groups of 4 people that have exactly 3 women. [2]

(ii) Calculate the number of different groups of at most 4 people where the number of women is the same as the number of men. [2]

13 **(a)** Eight books are to be arranged on a shelf. There are 4 mathematics books, 3 geography books and 1 French book.

(i) Find the number of different arrangements of the books if there are no restrictions. [1]

(ii) Find the number of different arrangements if the mathematics books have to be kept together. [3]

(iii) Find the number of different arrangements if the mathematics books have to be kept together and the geography books have to be kept together. [3]

(b) A team of 6 players is to be chosen from 8 men and 4 women. Find the number of different ways this can be done if

(i) there are no restrictions, [1]

(ii) there is at least one woman in the team. [2]

14 A 4-digit code is to be formed using 4 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find how many different codes can be formed if

(a) there are no restrictions, [1]

(b) only prime numbers are used, [1]

(c) two even numbers are followed by two odd numbers, [2]

(d) the code forms an even number. [2]

15 **(a)** Find the number of ways in which 12 people can be put into 3 groups containing 3, 4 and 5 people respectively. [3]

(b) 4-digit numbers are to be formed using four of the digits 2, 3, 7, 8 and 9. Each digit may be used once only in any 4-digit number. Find how many 4-digit numbers can be formed if

(i) there are no restrictions, [1]

(ii) the number is even, [1]

(iii) the number is greater than 7000 and odd. [3]

16 **(a)** In an examination, candidates must select 2 questions from the 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done. [2]

(b) The digits of the number 6 378 129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done. [2]

17 **(a)** **(i)** Find how many different 5-digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number. [1]

(ii) How many of the numbers found in **part (i)** are not divisible by 5? [1]

(iii) How many of the numbers found in **part (i)** are even and greater than 30 000? [4]

(b) The number of combinations of n items taken 3 at a time is 6 times the number of combinations of n items taken 2 at a time. Find the value of the constant n. [4]

18 **(a)** A 5-digit number is made using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. No digit may be used more than once in any 5-digit number. Find how many such 5-digit numbers are odd and greater than 70 000. [3]

(b) The number of combinations of n objects taken 3 at a time is 2 times the number of combinations of n objects taken 2 at a time. Find the value of n. [4]

19 **(a)** A 5-digit number is made using the digits 0, 1, 4, 5, 6, 7 and 9. No digit may be used more than once in any 5-digit number. Find how many such 5-digit numbers are even and greater than 50 000. [3]

(b) The number of combinations of n objects taken 4 at a time is equal to 6 times the number of combinations of n objects taken 2 at a time. Calculate the value of n. [5]

20 **(a)** A 5-digit number is to be formed from the digits 2, 5, 6, 7 and 9. Each digit may only be used once.

(i) Find the number of different 5-digit numbers that can be formed. [1]

(ii) Find the percentage of these numbers that are odd. [2]

(b) 12 people are placed at random in 3 groups of 4 people each. Find the number of ways that this can be done. [3]

21 **(a)** A six-character password is to be made from the following eight characters.

Digits	1	3	5	8	9
Symbols	*	$	#		

No character may be used more than once in a password.

Find the number of different passwords that can be chosen if

(i) there are no restrictions, [1]

(ii) the password starts with a digit and finishes with a digit, [2]

(iii) the password starts with three symbols. [2]

(b) The number of combinations of 5 objects selected from n objects is six times the number of combinations of 4 objects selected from $n-1$ objects. Find the value of n. [3]

22 **(a)** **(i)** Find how many different 5-digit numbers can be formed using the digits 1, 3, 5, 6, 8 and 9. No digit may be used more than once in any 5-digit number. [1]

(ii) How many of these 5-digit numbers are odd? [1]

(iii) How many of these 5-digit numbers are odd and greater than 60 000? [3]

(b) Given that $45 \times {}^{n}\mathrm{C}_{4} = (n+1) \times {}^{n+1}\mathrm{C}_{5}$, find the value of n. [4]

23 A 6-character password is to be formed from the following characters.

Letters	A	B	C	D
Numbers	1	2	3	4
Symbols	*	#	$	£

No character may be used more than once in any password.

(a) (i) Find the number of different 6-character passwords that can be formed. [1]

(ii) How many of these 6-character passwords end with a symbol? [1]

24 A group of 15 people includes 3 brothers. A team of 6 people is to be chosen from this group. The three brothers must not be separated. Find the number of possible teams that can be chosen. [3]

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25 A 4-digit code is to be formed using 4 different numbers selected from 2, 3, 4, 5, 6, 7, 8 and 9. Find how many possible codes there are if the code forms

(a) a number that is odd and greater than 5000, [3]

(b) a number greater than 5000 with a last digit that is prime. [3]

26 **(a)** **(i)** A 5-digit number is to be formed from the seven digits 0, 1, 2, 3, 4, 5, 6. Each digit can be used at most once in any number and the number does not start with 0. Find the number of ways in which this can be done. [2]

(ii) Find how many of these 5-digit numbers are even. [3]

(b) A team of 7 people is to be selected from a group of 9 women and 6 men. Find the number of different teams that can be selected which include at least one man. [2]

(c) **(i)** Show that ${}^nC_3 + {}^nC_2 = \frac{1}{6}(n^3 - n)$ for $n \geqslant 3$. [5]

(ii) Hence solve the equation ${}^nC_3 + {}^nC_2 = 4n$ where $n \geqslant 3$. [2]

27 **(a)** A team of 6 people is to be chosen from 10 people. Two of the people are sisters who must not be separated. Find the number of different teams that can be formed. [3]

(b) A 6-character password is to be chosen from the following characters.

Digits	2	4	8
Letters	*x*	*y*	*z*
Symbols	*	#	!

No character may be used more than once in any password. Find the number of different passwords that may be chosen if

(i) there are no other restrictions, [1]

(ii) the password starts with two letters and ends with two digits. [3]

28 **(a)** Find the number of ways in which 14 people can be put into 4 groups containing 2, 3, 4 and 5 people. [3]

(b) 6-digit numbers are to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit may be used only once in any 6-digit number. A 6-digit number must not start with 0. Find how many 6-digit numbers can be formed if

(i) there are no further restrictions [1]

(ii) the 6-digit number is divisible by 10 [1]

(iii) the 6-digit number is greater than 500 000 and even. [3]

29 **(a)** A team of 8 people is to be chosen from a group of 15 people.

(i) Find the number of different teams that can be chosen. [1]

(ii) Find the number of different teams that can be chosen if the group of 15 people contains a family of 4 people who must be kept together. [3]

(b) Given that $(n+9) \times {}^{n}\mathrm{P}_{10} = (n^2+243) \times {}^{n-1}\mathrm{P}_{9}$, find the value of n. [3]