1. 4037/11/M/J/16 Q5

Do not use a calculator in this question.

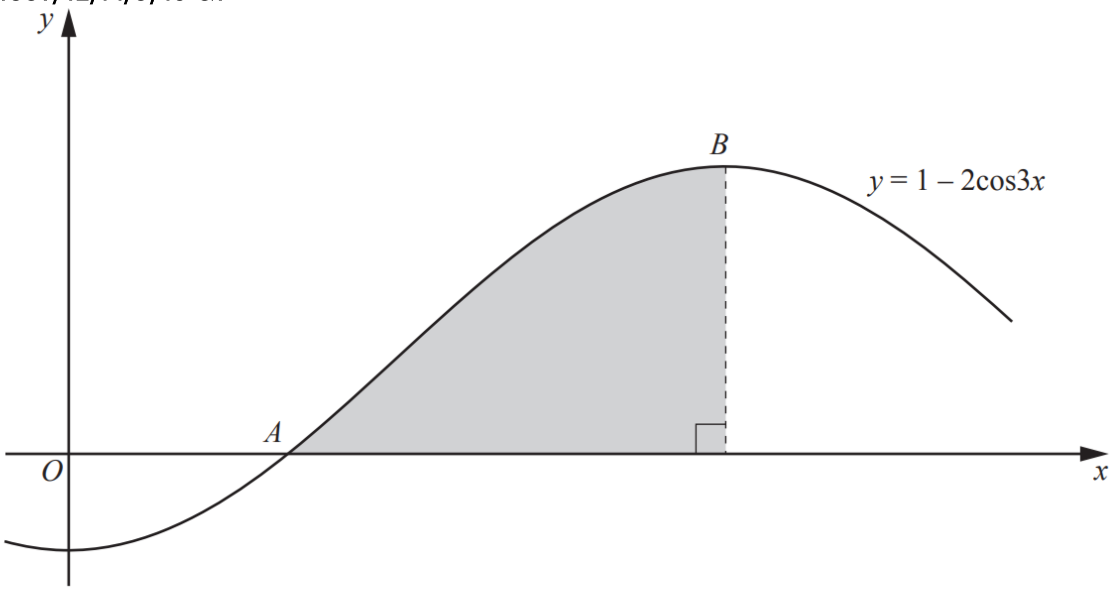
(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{e^{4x}}{4} - x e^{4x}\right) = pxe^{4x}$, where p is an integer to be found. [4]

(ii) Hence find the exact value of $\int_0^{\ln 2} xe^{4x}\,\mathrm{d}x$, giving your answer in the form $a\ln 2 + \frac{b}{c}$, where a, b and c are integers to be found. [4]

2. 4037/12/M/J/16 Q6

Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{3x}\sqrt{4x+1})$ can be written in the form $\frac{\mathrm{e}^{3x}(px+q)}{\sqrt{4x+1}}$, where p and q are integers to be found. [5]

3. 4037/12/M/J/16 Q7



The diagram shows part of the graph of $y = 1 - 2\cos 3x$, which crosses the x-axis at the point A and has a maximum at the point B.

(i) Find the coordinates of A. [2]

(ii) Find the coordinates of B. [2]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the perpendicular from B to the x-axis. [4]

4. 4037/12/M/J/16 Q9

A curve passes through the point $\left(2, -\frac{4}{3}\right)$ and is such that $\frac{dy}{dx} = (3x+10)^{-\frac{1}{2}}$.

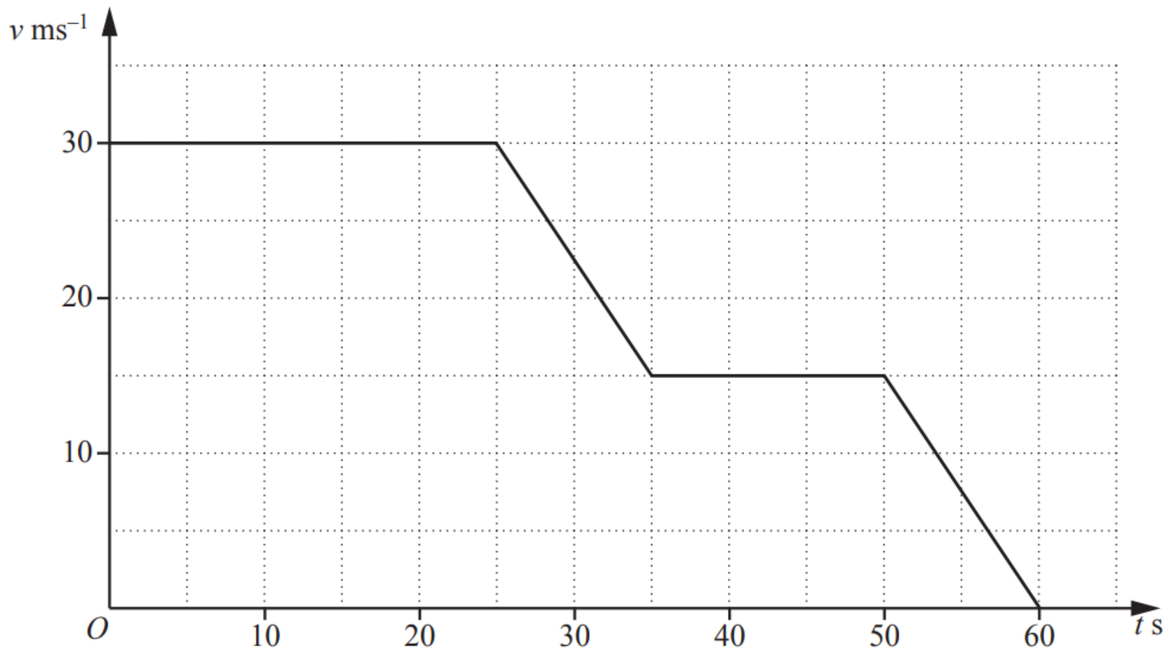
(i) Find the equation of the curve. [4]

The normal to the curve, at the point where $x = 5$, meets the line $y = -\frac{5}{3}$ at the point P.

(ii) Find the x-coordinate of P. [6]

5. 4037/12/M/J/16 Q11

(a)

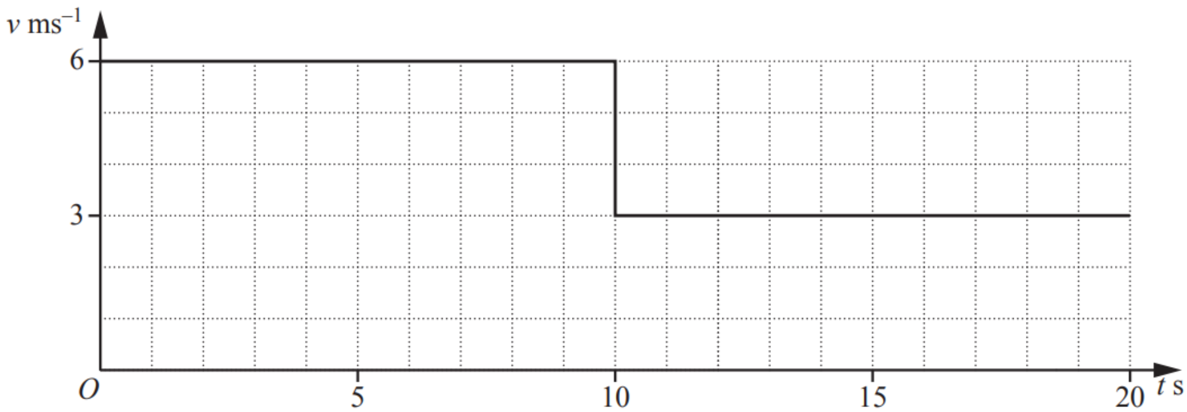


The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity $v\ \text{ms}^{-1}$ at time t s after leaving a fixed point.

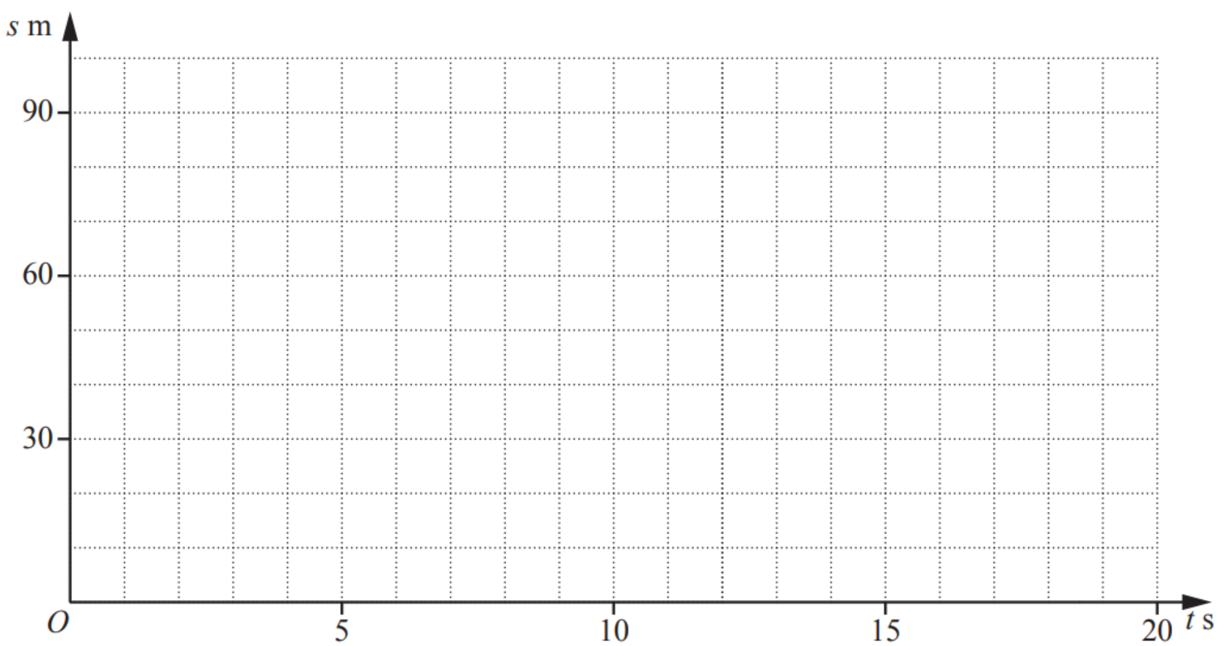
(i) Find the distance travelled by the particle P. [2]

(ii) Write down the deceleration of the particle when $t = 30$. [1]

(b) The diagram shows a velocity-time graph of a particle Q moving in a straight line with velocity v ms^{-1}, at time t s after leaving a fixed point.



The displacement of Q at time t s is s m. On the axes below, draw the corresponding displacement-time graph for Q. [2]



(c) The velocity, v ms^{-1}, of a particle R moving in a straight line, t s after passing through a fixed point O, is given by $v = 4e^{2t} + 6$.

(i) Explain why the particle is never at rest. [1]

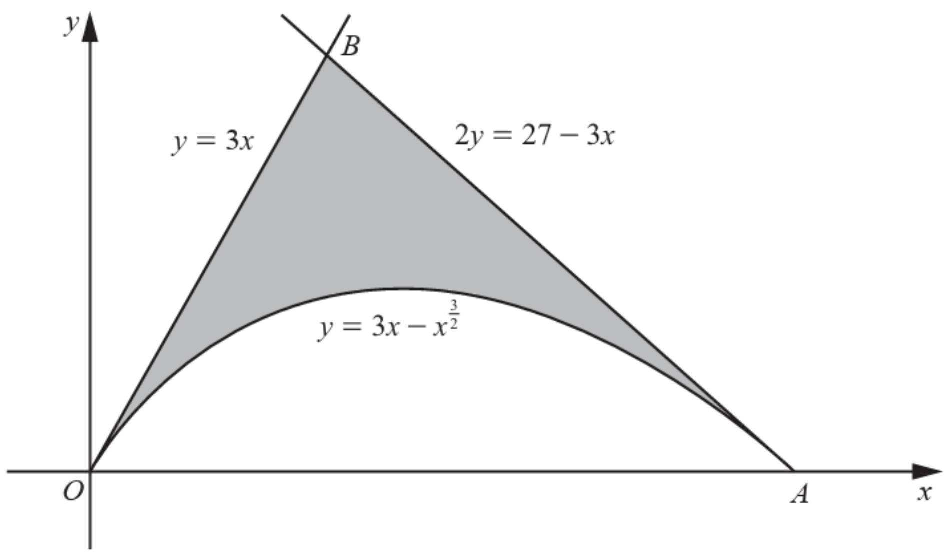
(ii) Find the exact value of t for which the acceleration of R is 12 ms^{-2}. [2]

(iii) Showing all your working, find the distance travelled by R in the interval between $t = 0.4$ and $t = 0.5$. [4]

6. 4037/21/M/J/16 Q11

(i) Find $\int (3x - x^{\frac{3}{2}})\,dx$. [2]

The diagram shows part of the curve $y = 3x - x^{\frac{3}{2}}$ and the lines $y = 3x$ and $2y = 27 - 3x$. The curve and the line $y = 3x$ meet the x-axis at O and the curve and the line $2y = 27 - 3x$ meet the x-axis at A.



(ii) Find the coordinates of A. [1]

(iii) Verify that the coordinates of B are $(3, 9)$. [1]

(iv) Find the area of the shaded region. [4]

7. 4037/21/M/J/16 Q12

A curve has equation $y = \dfrac{2x-5}{x-1} - 12x$.

(i) Find $\dfrac{dy}{dx}$. [3]

(ii) Find $\dfrac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary points of the curve and determine their nature. [5]

8. 4037/22/M/J/16 Q2

Variables x and y are related by the equation $y = \dfrac{5x-1}{3-x}$.

(i) Find $\dfrac{dy}{dx}$, simplifying your answer. [2]

(ii) Hence find the approximate change in x when y increases from 9 by the small amount 0.07. [3]

9. 4037/22/M/J/16 Q9

(a) Find $\int \frac{x^3 + x^2 + 1}{x^2} \, dx$. [3]

(b) **(i)** Find $\int \sin(5x + \pi) \, dx$. [2]

(ii) Hence evaluate $\int_{-\frac{\pi}{5}}^{0} \sin(5x + \pi) \, dx$. [2]

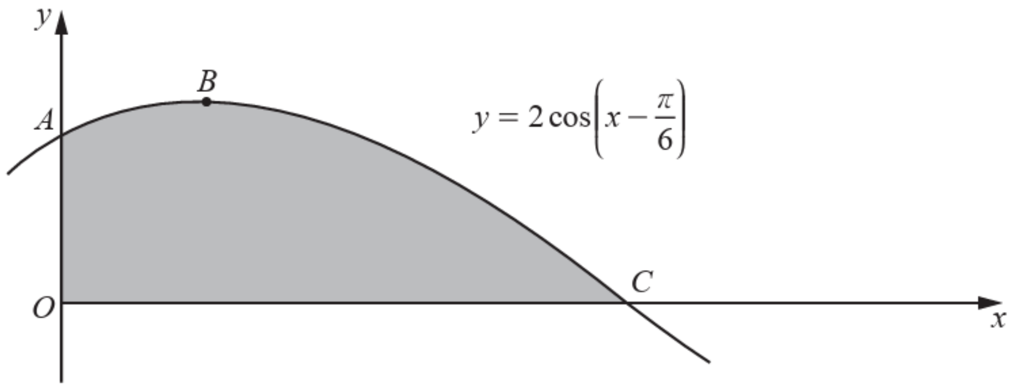
10. 4037/12/O/N/16 Q5

(i) Find the equation of the normal to the curve $y = \frac{1}{2}\ln(3x+2)$ at the point P where $x = -\frac{1}{3}$. [4]

The normal to the curve at the point P intersects the y-axis at the point Q. The curve $y = \frac{1}{2}\ln(3x+2)$ intersects the y-axis at the point R.

(ii) Find the area of the triangle PQR. [3]

11. 4037/12/O/N/16 Q7



The diagram shows part of the graph of $y = 2\cos\left(x - \frac{\pi}{6}\right)$. The graph intersects the y-axis at the point A, has a maximum point at B and intersects the x-axis at the point C.

(i) Find the coordinates of A. [1]

(ii) Find the coordinates of B. [2]

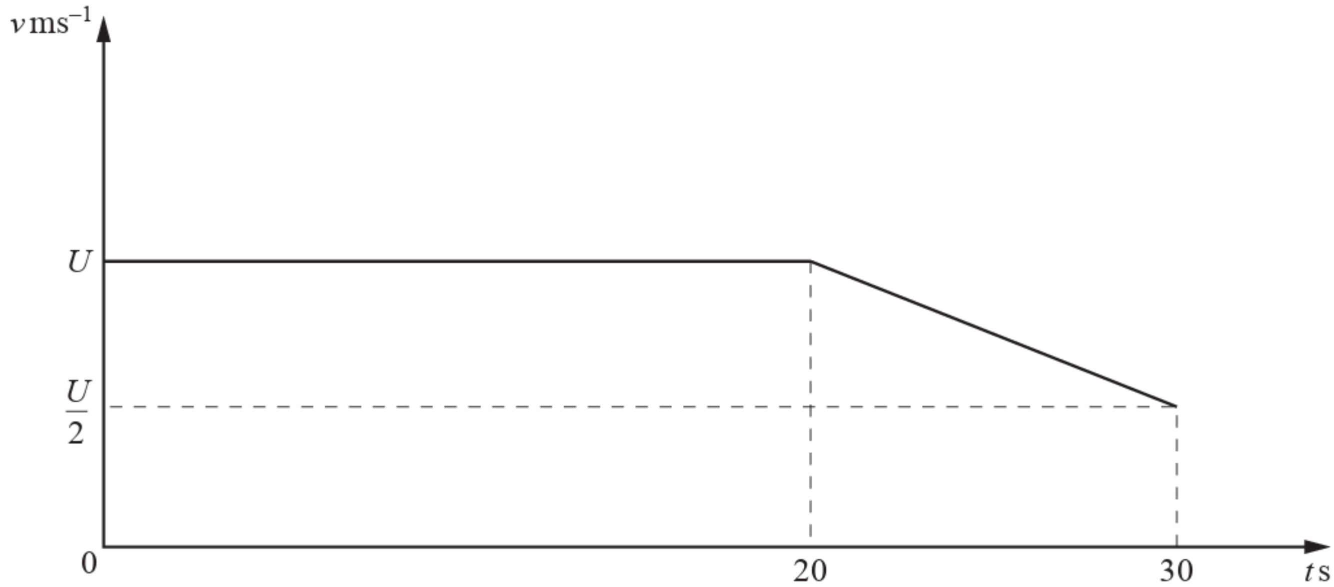
(iii) Find the coordinates of C. [2]

(iv) Find $\int 2\cos\left(x - \frac{\pi}{6}\right)\mathrm{d}x$. [1]

(v) Hence find the area of the shaded region. [2]

12. 4037/12/O/N/16 Q10

(a)



The diagram shows part of the velocity-time graph for a particle, moving at $v\,\text{ms}^{-1}$ in a straight line, t s after passing through a fixed point. The particle travels at $U\,\text{ms}^{-1}$ for 20 s and then decelerates uniformly for 10 s to a velocity of $\frac{U}{2}\text{ms}^{-1}$. In this 30 s interval, the particle travels 165 m.

(i) Find the value of U. [3]

(ii) Find the acceleration of the particle between $t = 20$ and $t = 30$. [2]

(b) A particle P travels in a straight line such that, t s after passing through a fixed point O, its velocity, $v\,\text{ms}^{-1}$, is given by $v = \left(e^{\frac{t^2}{8}} - 4\right)^3$.

(i) Find the speed of P at O. [1]

(ii) Find the value of t for which P is instantaneously at rest. [2]

(iii) Find the acceleration of P when $t = 1$. [4]

13. 4037/13/O/N/16 Q6

(i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x^2 - 11))$. [2]

(ii) Hence show that $\int \frac{x}{3x^2 - 11} \mathrm{d}x = p\ln(3x^2 - 11) + c$, where p is a constant to be found, and c is a constant of integration. [1]

(iii) Given that $\int_2^a \frac{x}{3x^2 - 11} \mathrm{d}x = \ln 2$, where $a > 2$, find the value of a. [4]

14. 4037/13/O/N/16 Q10

A curve $y = f(x)$ is such that $f'(x) = 6x - 8e^{2x}$.

(i) Given that the curve passes through the point $P\ (0, -3)$, find the equation of the curve. [5]

The normal to the curve $y = f(x)$ at P meets the line $y = 2 - 3x$ at the point Q.

(ii) Find the area of the triangle OPQ, where O is the origin. [5]

15. 4037/13/O/N/16 Q11

A particle moving in a straight line has a velocity of $v\ \text{ms}^{-1}$ such that, t s after leaving a fixed point, $v = 4t^2 - 8t + 3$.

(i) Find the acceleration of the particle when $t = 3$. [2]

(ii) Find the values of t for which the particle is momentarily at rest. [2]

(iii) Find the total distance the particle has travelled when $t = 1.5$. [5]

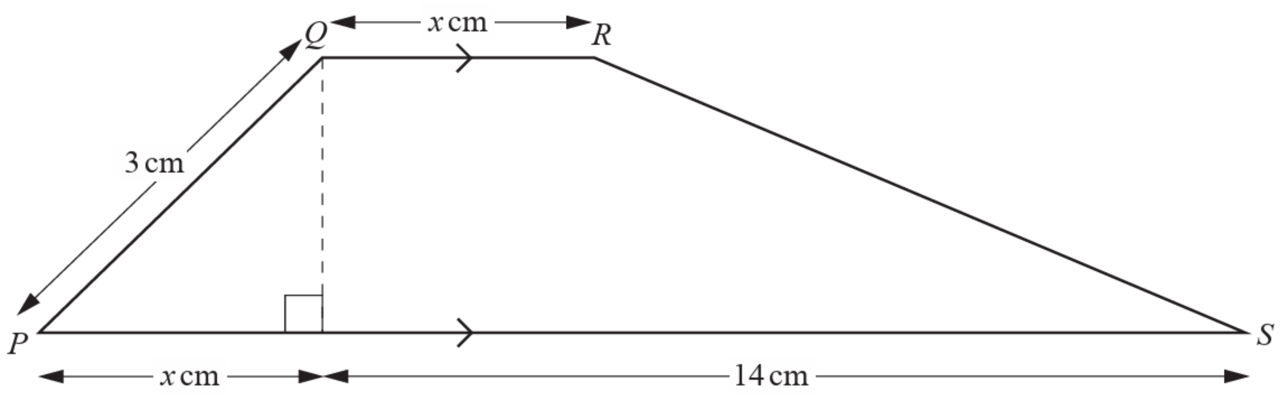
16. 4037/22/O/N/16 Q5

The curve with equation $y = x^3 + 2x^2 - 7x + 2$ passes through the point A (−2, 16). Find

(i) the equation of the tangent to the curve at the point A, [3]

(ii) the coordinates of the point where this tangent meets the curve again. [5]

17. 4037/22/O/N/16 Q7



(i) Show that the area, $A\,\text{cm}^2$, of the trapezium $PQRS$ is given by $A = (7 + x)\sqrt{9 - x^2}$. [2]

(ii) Given that x can vary, find the stationary value of A. [7]

18. 4037/22/O/N/16 Q8

The function f(x) is given by $\quad \mathrm{f}(x) = \dfrac{3x^3 - 1}{x^3 + 1} \quad$ for $\quad 0 \leqslant x \leqslant 3$.

(i) Show that $\mathrm{f}'(x) = \dfrac{kx^2}{(x^3 + 1)^2}$, where k is a constant to be determined. [3]

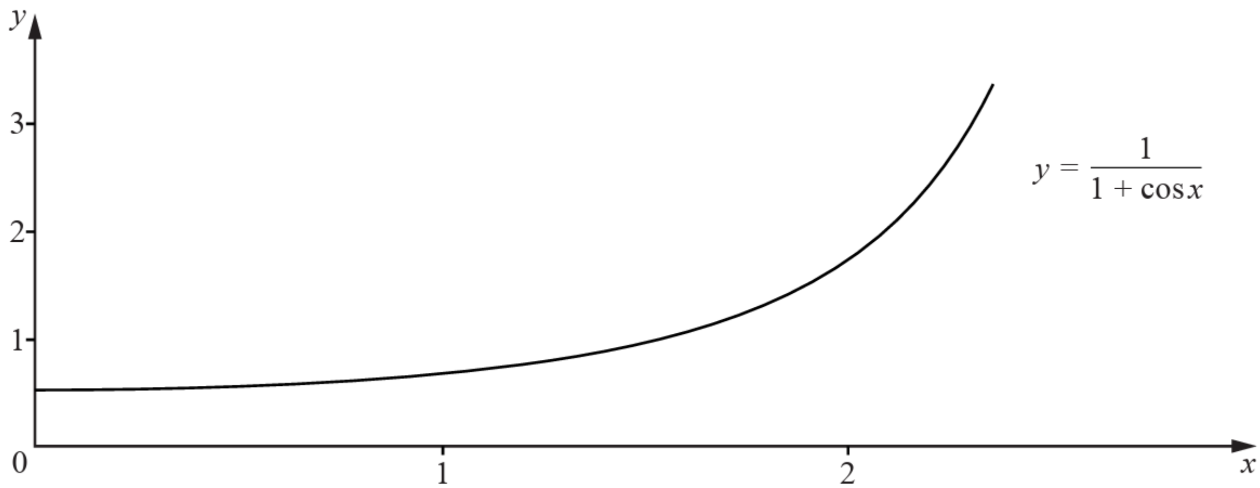
(ii) Find $\displaystyle\int \frac{x^2}{(x^3 + 1)^2}\,\mathrm{d}x$ and hence evaluate $\displaystyle\int_1^2 \frac{x^2}{(x^3 + 1)^2}\,\mathrm{d}x$. [4]

(iii) Find $f^{-1}(x)$, stating its domain. [4]

19. 4037/23/O/N/16 Q3

(i) Show that $\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) = \frac{1}{1+\cos x}$. [4]

(ii)



The diagram shows part of the graph of $y = \dfrac{1}{1+\cos x}$. Use the result from part **(i)** to find the area enclosed by the graph and the lines $x = 0$, $x = 2$ and $y = 0$. [2]

20. 4037/11/M/J/17 Q7

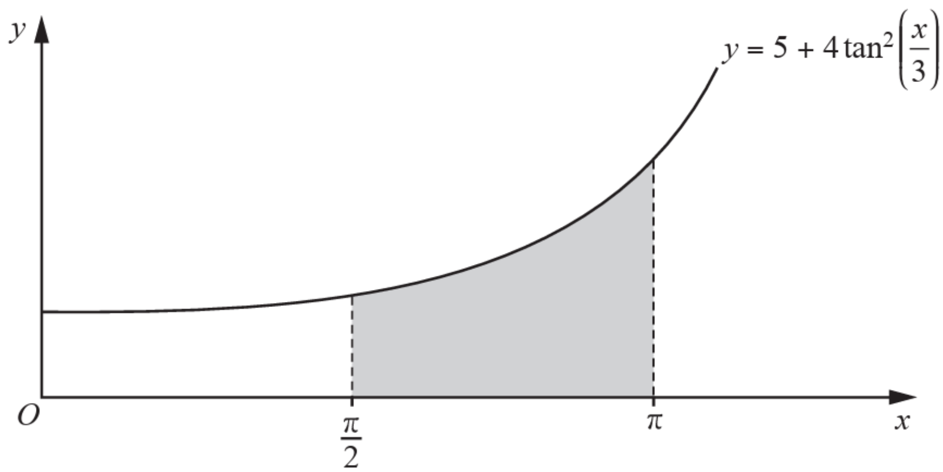
Show that the curve $y = (3x^2 + 8)^{\frac{5}{3}}$ has only one stationary point. Find the coordinates of this stationary point and determine its nature. [8]

21. 4037/11/M/J/17 Q9

(i) Show that $5 + 4\tan^2\left(\frac{x}{3}\right) = 4\sec^2\left(\frac{x}{3}\right) + 1.$ [1]

(ii) Given that $\frac{d}{dx}\left(\tan\left(\frac{x}{3}\right)\right) = \frac{1}{3}\sec^2\left(\frac{x}{3}\right)$, find $\int \sec^2\left(\frac{x}{3}\right) dx.$ [1]

(iii)



The diagram shows part of the curve $y = 5 + 4\tan^2\left(\frac{x}{3}\right)$. Using the results from parts **(i)** and **(ii)**, find the exact area of the shaded region enclosed by the curve, the x-axis and the lines $x = \frac{\pi}{2}$ and $x = \pi$. [5]

22. 4037/11/M/J/17 Q10

(a) Given that $y = \frac{e^{3x}}{4x^2 + 1}$, find $\frac{dy}{dx}$. [3]

(b) Variables x, y and t are such that $y = 4\cos\left(x + \frac{\pi}{3}\right) + 2\sqrt{3}\sin\left(x + \frac{\pi}{3}\right)$ and $\frac{dy}{dt} = 10$.

(i) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$. [3]

(ii) Find the value of $\frac{dx}{dt}$ when $x = \frac{\pi}{2}$. [2]

23. 4037/12/M/J/17 Q2

It is given that $y = \frac{(5x^2 + 4)^{\frac{1}{2}}}{x + 1}$. Showing all your working, find the exact value of $\frac{dy}{dx}$ when $x = 3$. [5]

24. 4037/12/M/J/17 Q11

The curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{2}, \frac{7}{2}\right)$ and is such that $\mathrm{f}'(x) = \mathrm{e}^{2x-1}$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $\mathrm{f}''(x) = 4$, giving your answer in the form $a + b\ln\sqrt{2}$, where a and b are constants. [4]

25. 4037/21/M/J/17 Q1

Find the equation of the curve which passes through the point (2, 17) and for which $\frac{dy}{dx} = 4x^3 + 1$. [4]

26. 4037/21/M/J/17 Q3

The variables x and y are such that $y = \ln(x^2 + 1)$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) Hence, find the approximate change in y when x increases from 3 to $3 + h$, where h is small. [2]

27. 4037/21/M/J/17 Q12

A particle moves in a straight line so that, t seconds after passing a fixed point O, its displacement, s m, from O is given by

$$s = 1 + 3t - \cos 5t.$$

(i) Find the distance between the particle's first two positions of instantaneous rest. [7]

(ii) Find the acceleration when $t = \pi$. [2]

28. 4037/22/M/J/17 Q4

The point P lies on the curve $y = 3x^2 - 7x + 11$. The normal to the curve at P has equation $5y + x = k$. Find the coordinates of P and the value of k. [6]

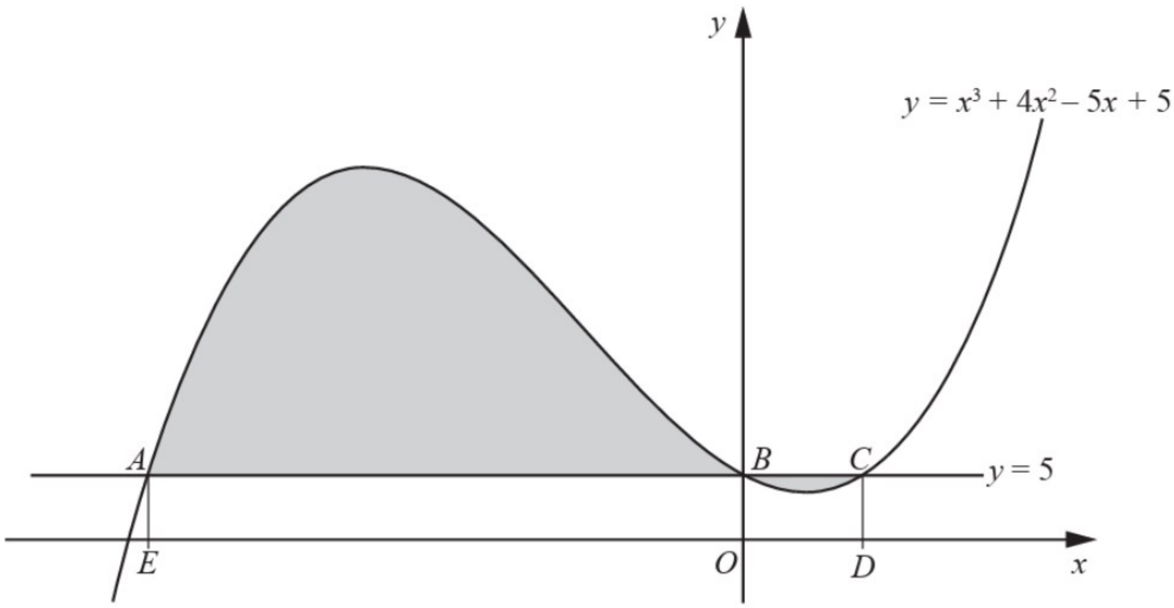
29. 4037/22/M/J/17 Q5

(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}[0.4x^5(0.2 - \ln 5x)] = kx^4 \ln 5x$, where k is an integer to be found. [2]

(ii) Express $\ln 125x^3$ in terms of $\ln 5x$. [1]

(iii) Hence find $\int (x^4 \ln 125x^3)\,\mathrm{d}x$. [2]

30. 4037/22/M/J/17 Q11



The diagram shows part of the curve $y = x^3 + 4x^2 - 5x + 5$ and the line $y = 5$. The curve and the line intersect at the points A, B and C. The points D and E are on the x-axis and the lines AE and CD are parallel to the y-axis.

(i) Find $\int (x^3 + 4x^2 - 5x + 5)\,dx$. [2]

(ii) Find the area of each of the rectangles $OEAB$ and $OBCD$. [4]

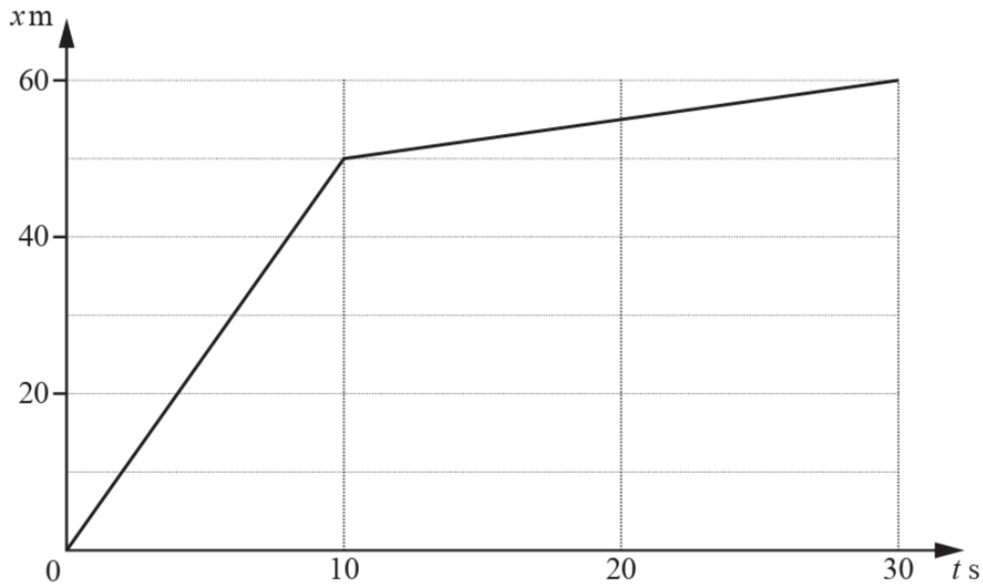
(iii) Hence calculate the total area of the shaded regions enclosed between the line and the curve. You must show all your working. [4]

31. 4037/12/O/N/17 Q4

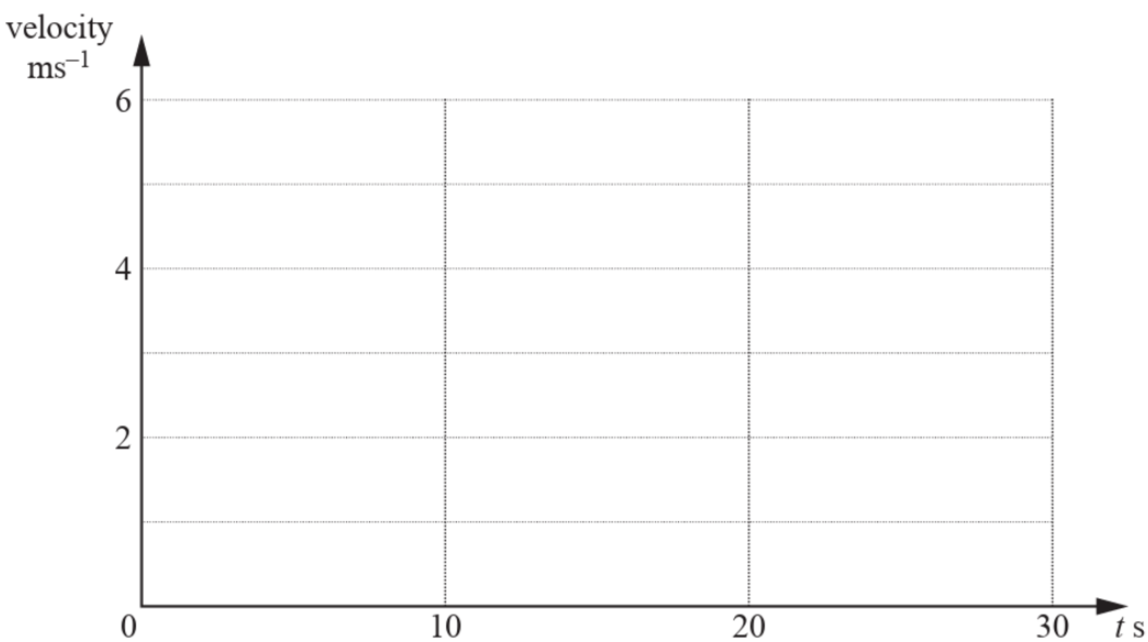
Given that $y = \dfrac{\ln(3x^2 + 2)}{x^2 + 1}$, find the value of $\dfrac{dy}{dx}$ when $x = 2$, giving your answer as $a + b\ln 14$, where a and b are fractions in their simplest form. [6]

32. 4037/12/O/N/17 Q9

(a)



The diagram shows the displacement-time graph of a particle P which moves in a straight line such that, t s after leaving a fixed point O, its displacement from O is x m. On the axes below, draw the velocity-time graph of P.



[3]

(b) A particle Q moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, t s after passing through a fixed point O, is given by $v = 3\mathrm{e}^{-5t} + \dfrac{3t}{2}$, for $t \geqslant 0$.

(i) Find the velocity of Q when $t = 0$. [1]

(ii) Find the value of t when the acceleration of Q is zero. [3]

(iii) Find the distance of Q from O when $t = 0.5$. [4]

33. 4037/13/O/N/17 Q2

A curve is such that its gradient at the point (x, y) is given by $10e^{5x} + 3$. Given that the curve passes though the point $(0, 9)$, find the equation of the curve. [4]

34. 4037/13/O/N/17 Q5

(i) Find $\int (7x - 10)^{-\frac{3}{5}} \mathrm{d}x$. [2]

(ii) Given that $\int_6^a (7x - 10)^{-\frac{3}{5}} \mathrm{d}x = \frac{25}{14}$, find the exact value of a. [3]

35. 4037/13/O/N/17 Q8

It is given that $y = (x-4)(3x-1)^{\frac{5}{3}}$.

(i) Show that $\dfrac{dy}{dx} = (3x-1)^{\frac{2}{3}}(Ax+B)$, where A and B are integers to be found. [5]

(ii) Hence find, in terms of h, where h is small, the approximate change in y when x increases from 3 to $3+h$. [3]

36. 4037/22/O/N/17 Q6

The volume of a closed cylinder of base radius x cm and height h cm is $500\,\text{cm}^3$.

(i) Express h in terms of x. [1]

(ii) Show that the total surface area of the cylinder is given by $A = 2\pi x^2 + \dfrac{1000}{x}\,\text{cm}^2$. [2]

(iii) Given that x can vary, find the stationary value of A and show that this value is a minimum. [5]

37. 4037/22/O/N/17 Q7

The gradient of the normal to a curve at the point with coordinates (x, y) is given by $\frac{\sqrt{x}}{1-3x}$.

(i) Find the equation of the curve, given that the curve passes through the point $(1, -10)$. [5]

(ii) Find, in the form $y = mx + c$, the equation of the tangent to the curve at the point where $x = 4$. [4]

38. 4037/22/O/N/17 Q9

(i) Find $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)$. [2]

(ii) Hence find $\int \ln x \, \mathrm{d}x$. [2]

(iii) Hence, given that $k > 0$, show that $\int_k^{2k} \ln x \, \mathrm{d}x = k(\ln 4k - 1)$. [4]

39. 4037/22/O/N/17 Q10 (ii), (iii), (iv)

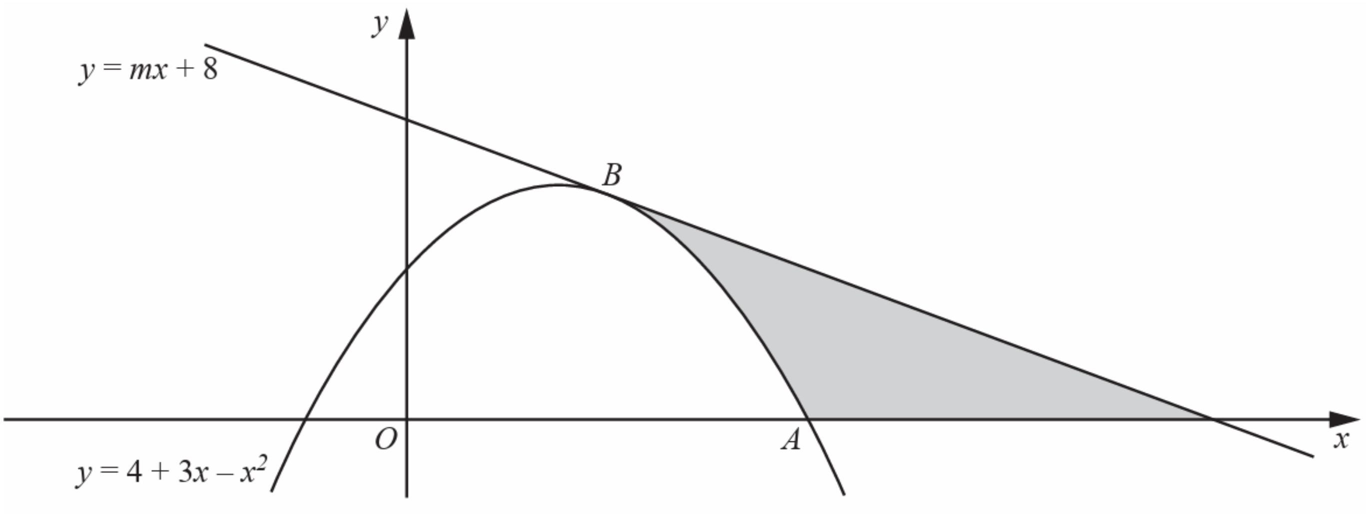
It is given that $y = \tan x + 6\sin x$.

(ii) Find $\frac{dy}{dx}$. [2]

(iii) If $\frac{dy}{dx} = 7$ show that $6\cos^3 x - 7\cos^2 x + 1 = 0$. [2]

(iv) Hence solve the equation $\frac{dy}{dx} = 7$ for $0 \leqslant x \leqslant \pi$ radians. [2]

40. 4037/22/O/N/17 Q11



The diagram shows the curve $y = 4 + 3x - x^2$ intersecting the positive x-axis at the point A. The line $y = mx + 8$ is a tangent to the curve at the point B. Find

(i) the coordinates of A, [2]

(ii) the value of m, [3]

(iii) the coordinates of B, [2]

(iv) the area of the shaded region, showing all your working. [5]

41. 4037/23/O/N/17 Q7

A particle moving in a straight line passes through a fixed point O. Its velocity, v ms^{-1}, t s after passing through O, is given by $v = 3\cos 2t - 1$ for $t \geqslant 0$.

(i) Find the value of t when the particle is first at rest. [2]

(ii) Find the displacement from O of the particle when $t = \frac{\pi}{4}$. [3]

(iii) Find the acceleration of the particle when it is first at rest. [3]

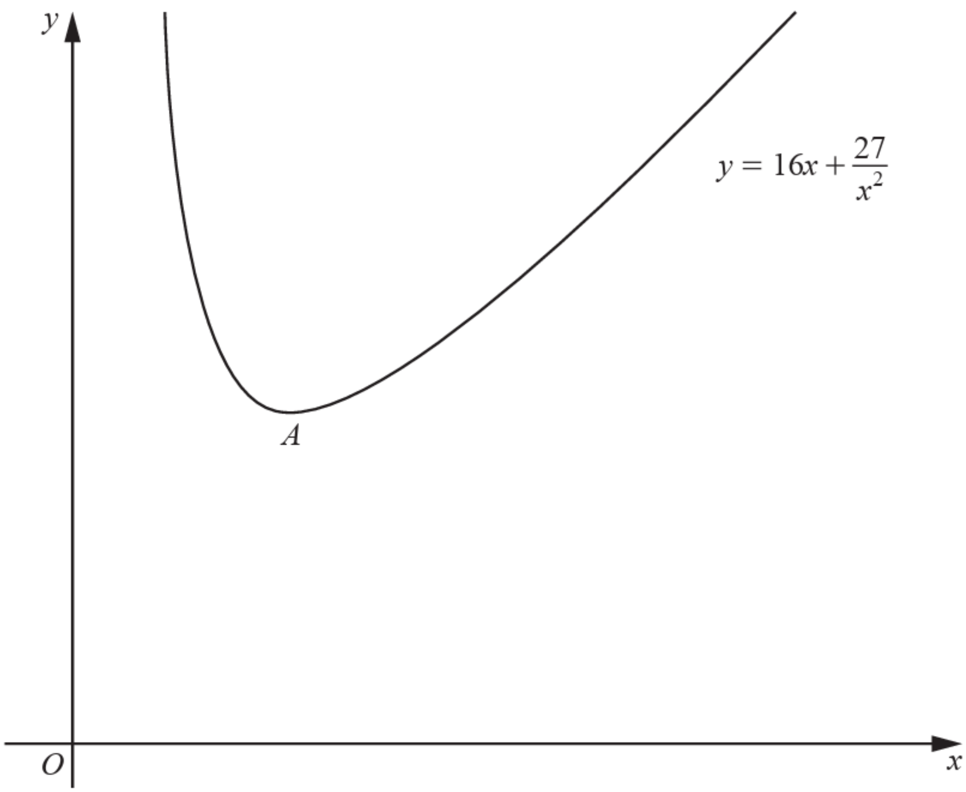
42. 4037/23/O/N/17 Q9

(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{x^3}\right) = \frac{1 - 3\ln x}{x^4}$. [3]

(ii) Find the exact coordinates of the stationary point of the curve $y = \frac{\ln x}{x^3}$. [3]

(iii) Use the result from part **(i)** to find $\int\left(\frac{\ln x}{x^4}\right)\mathrm{d}x$. [4]

43. 4037/11/M/J/18 Q11



The diagram shows part of the graph of $y = 16x + \frac{27}{x^2}$, which has a minimum at A.

(i) Find the coordinates of A. [4]

The points P and Q lie on the curve $y = 16x + \dfrac{27}{x^2}$ and have x-coordinates 1 and 3 respectively.

(ii) Find the area enclosed by the curve and the line PQ. You must show all your working. [6]

44. 4037/11/M/J/18 Q12

A curve is such that $\dfrac{d^2y}{dx^2} = (2x-5)^{-\frac{1}{2}}$. Given that the curve has a gradient of 6 at the point $\left(\dfrac{9}{2}, \dfrac{2}{3}\right)$, find the equation of the curve. [8]

45. 4037/12/M/J/18 Q4

A particle P moves so that its displacement, x metres from a fixed point O, at time t seconds, is given by $x = \ln(5t + 3)$.

(i) Find the value of t when the displacement of P is 3m. [2]

(ii) Find the velocity of P when $t = 0$. [2]

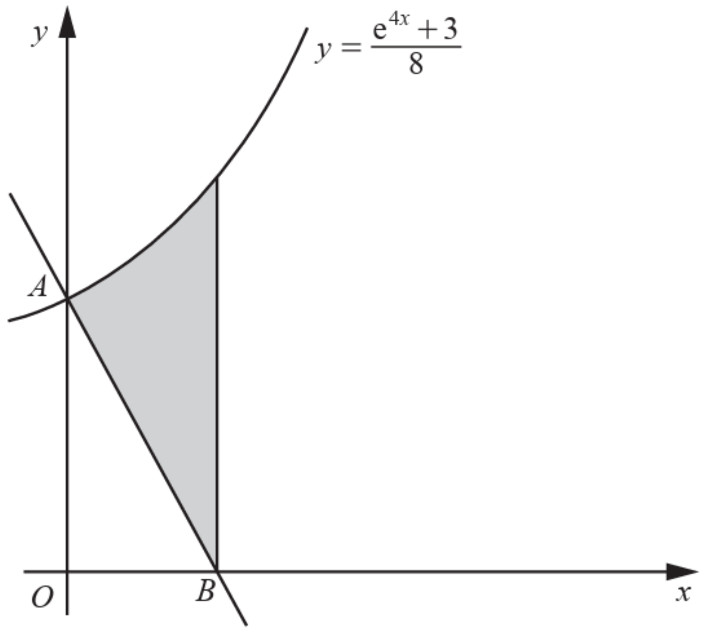
(iii) Explain why, after passing through O, the velocity of P is never negative. [1]

(iv) Find the acceleration of P when $t = 0$. [2]

46. 4037/12/M/J/18 Q6

Find the coordinates of the stationary point of the curve $y = \dfrac{x+2}{\sqrt{2x-1}}$. [6]

47. 4037/12/M/J/18 Q11



The diagram shows the graph of the curve $y = \frac{e^{4x} + 3}{8}$. The curve meets the y-axis at the point A.

The normal to the curve at A meets the x-axis at the point B. Find the area of the shaded region enclosed by the curve, the line AB and the line through B parallel to the y-axis. Give your answer in the form $\frac{e}{a}$, where a is a constant. You must show all your working.

[10]

48. 4037/21/M/J/18 Q2

The variables x and y are such that $y = \ln(3x - 1)$ for $x > \frac{1}{3}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Hence find the approximate change in x when y increases from $\ln(1.2)$ to $\ln(1.2) + 0.125$. [3]

49. 4037/21/M/J/18 Q7

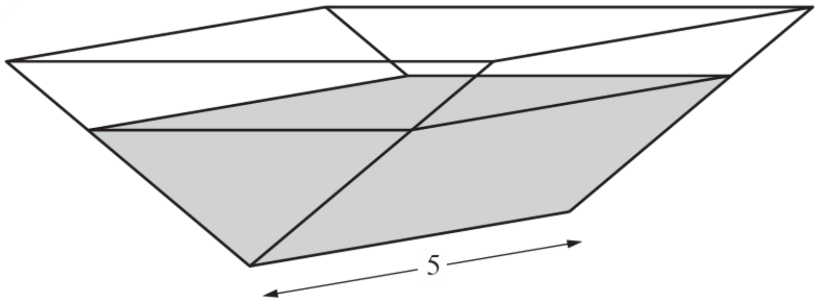
Differentiate with respect to x

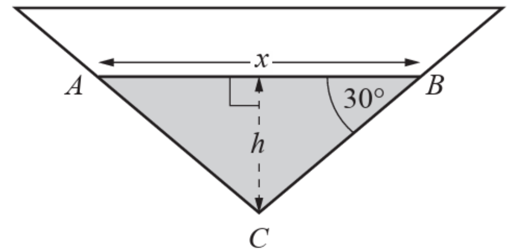
(i) $4x\tan x$, [2]

(ii) $\dfrac{e^{3x+1}}{x^2-1}$. [3]

50. 4037/21/M/J/18 Q12

In this question all lengths are in metres.





A water container is in the shape of a triangular prism. The diagrams show the container and its cross-section. The cross-section of the water in the container is an isosceles triangle ABC, with angle ABC = angle $BAC = 30°$. The length of AB is x and the depth of water is h. The length of the container is 5.

(i) Show that $x = 2\sqrt{3}h$ and hence find the volume of water in the container in terms of h. [3]

(ii) The container is filled at a rate of $0.5\,\text{m}^3$ per minute. At the instant when h is 0.25 m, find

(a) the rate at which h is increasing, [4]

(b) the rate at which x is increasing. [2]