1.

$\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = e^{4x} - \left(\left(x \times 4e^{4x}\right) + e^{4x}\right)$	**B1**	for $\frac{d}{dx}\left(\frac{e^{4x}}{4}\right) = e^{4x}$
	M1	for attempt to differentiate a product
	A1	for a correct product
$= -4xe^{4x}$	**A1**	for correct final answer
$\int_0^{\ln 2} xe^{4x}\ dx = -\frac{1}{4}\left[\frac{e^{4x}}{4} - xe^{4x}\right]_0^{\ln 2}$	**B1FT**	**FT** for use of *their* $\frac{1}{p} \times \left(\frac{e^{4x}}{4} - xe^{4x}\right)$, must be numerical *p*, but $\neq 0$
$= -\frac{1}{4}\left(\left(\frac{16}{4} - 16\ln 2\right) - \frac{1}{4}\right)$	**B1**	for $e^{4\ln 2} = 16$
	M1	for correct use of limits, must be an integral of the correct form
$= 4\ln 2 - \frac{15}{16}$	**A1**	

2.

6	$\frac{d}{dx}\left(e^{3x}(4x+1)^{\frac{1}{2}}\right)$		
	$= e^{3x}\frac{1}{2} \times 4(4x+1)^{-\frac{1}{2}} + 3e^{3x}(4x+1)^{\frac{1}{2}}$	**B1**	for $re^{3x}(4x+1)^{-\frac{1}{2}}$ must be part of a sum, $r = \frac{1}{2}$ or 2 or $\frac{1}{2} \times 4$
		B1	for $se^{3x}(4x+1)^{\frac{1}{2}}$ must be part of a sum, s is 1 or 3
	$= \frac{2e^{3x}}{(4x+1)^{\frac{1}{2}}} + 3e^{3x}(4x+1)^{\frac{1}{2}}$	**B1**	for all correct, allow unsimplified
	$= \frac{e^{3x}}{(4x+1)^{\frac{1}{2}}}(2 + 12x + 3)$	**DM1**	for $\frac{e^{3x}}{(4x+1)^{\frac{1}{2}}}(a + bx)$, dependent on first 2 **B** marks , must be using a correct method, collecting terms in the numerator correctly
	$= \frac{e^{3x}}{(4x+1)^{\frac{1}{2}}}(12x + 5)$	**A1**	

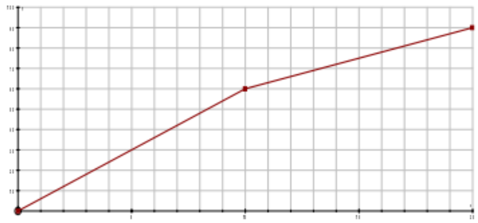
3.

7 (i)	$\cos 3x = \frac{1}{2}$, $x = \frac{\pi}{9}$ or 0.349, 20°, allow 0.35	**M1** **A1**	for correct attempt to solve the trigonometric equation
(ii)	$B\left(\frac{\pi}{3}, 3\right)$ or $(1.05, 3)$, $(60°, 3)$	**B1B1**	**B1** for each, must be in correct position or in terms of $x =$ and $y =$
(iii)	$\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} 1 - 2\cos 3x \, dx = \left[x - \frac{2}{3}\sin 3x\right]_{\frac{\pi}{9}}^{\frac{\pi}{3}}$	**M1** **A1**	for $x \pm a\sin 3x$ attempt to integrate at least one term for correct integration
	$= \frac{\pi}{3} - \left(\frac{\pi}{9} - \left(\frac{2}{3} \times \frac{\sqrt{3}}{2}\right)\right)$	**DM1**	for correct use of limits from **(i)** and **(ii)**, must be in radians
	$= \frac{2\pi}{9} + \frac{\sqrt{3}}{3}$ oe or 1.28	**A1**	

4.

9 (i)	$y = \frac{2}{3}(3x+10)^{\frac{1}{2}} \quad (+c)$	**B1**	for $p(3x+10)^{\frac{1}{2}}$ where p is a constant
		B1	for $\frac{2}{3}(3x+10)^{\frac{1}{2}}$ oe unsimplified
	passes through $\left(2, -\frac{4}{3}\right)$, so $c = -4$	**M1**	for attempt to find c, must have attempt to integrate, must have the first **B1**
	$y = \frac{2}{3}(3x+10)^{\frac{1}{2}} - 4$ oe	**A1**	
(ii)	When $x = 5$, $y = -\frac{2}{3}$	**B1**	
	perpendicular gradient $= -5$	**B1**	
		M1	for attempt at the normal using *their* perpendicular gradient and *their y* value (but not $y = -\frac{4}{3}$ or $-\frac{5}{3}$).
	Equation of normal: $y + \frac{2}{3} = -5(x-5)$	**A1**	
	When $y = -\frac{5}{3}$,	**DM1**	for use of $y = -\frac{5}{3}$ in their normal equation to get as far as $x = \ldots$
	$x = 5.2$ oe	**A1**	

5.

11 (a) (i)	Distance = area under graph	**M1**	for attempt to find the area, one correct area seen (triangle, rectangle or trapezium) as part of a sum.
	$= 1275$	**A1**	
(ii)	deceleration is 1.5 oe	**B1**	
(b)		**B1**	for a straight line between $(0,0)$ and $(10,60)$
		B1FT	**FT** a straight line between $(10, 60)$ and $(20, 90)$, a displacement vector $\begin{pmatrix}10\\30\end{pmatrix}$ from *their* $(10, \textit{their}\,60)$
(c) (i)	e^{2t} is always positive or oe	**B1**	
(ii)	$a = 8e^{2t}$ $e^{2t} = \frac{3}{2}$	**M1**	for attempt to differentiate, must be of the form pe^{2t}, equate to 12 and solve.
	$t = \frac{1}{2}\ln\frac{3}{2}$, $\ln\sqrt{\frac{3}{2}}$ or $\frac{1}{2}\ln 1.5$	**A1**	Allow fractions equivalent to $\frac{3}{2}$
(iii)	$s = \left[2e^{2t} + 6t\right]_{0.4}^{0.5}$	**M1** **A1**	for attempt to integrate to get $qe^{2t} + 6t$ all correct
	$= (2e + 3) - (2e^{0.8} + 2.4)$ $(= 8.436 - 6.851)$	**DM1**	for correct use of limits or considering distances separately, ignore attempts at c
	$= 1.59$, allow 1.58	**A1**	

6.

11 (i)	$\frac{3x^2}{2} - \frac{2x^{5/2}}{5}$ $(+c)$ isw	**B1+B1**	
(ii)	(9, 0) oe	**B1**	Not just $x = 9$
(iii)	Substitute (3, 9) into **both** lines Or solves simultaneously ($6x = 27 - 3x$ oe) to get $x = 3, y = 9$	**B1**	$3 \times 3 = 9$ **and** $\frac{27 - 3\times 3}{2} = 9$
(iv)	[Area $AOB =$] $\frac{1}{2}\times 9\times 9$ oe $\quad (\frac{81}{2}$ or 40.5)	**M1**	Uses *their* (ii). May split into 2 triangles (13.5 and 27). May integrate. Must be a complete method.
	their $\left[\frac{3(9)^2}{2} - \frac{2(9)^{5/2}}{5}\right] - [0]$ $\quad$ (= 24.3)	**M1**	lower limit may be omitted but must be correct if seen
	their $\frac{81}{2}$ – *their* $\frac{243}{10}$	**M1**	must be from genuine attempts at area of triangle and area under curve
	16.2	**A1**	

7.

12 (i)	$\left[\frac{dy}{dx} =\right] \frac{2(x-1)-(2x-5)}{(x-1)^2}$	**M1A1**	Allow slips in $\frac{du}{dx}$ and $\frac{dv}{dx}$ but must be explicit. Allow $(x-1)^2 = x^2 - 2x + 1$
	– 12 isw	**B1**	
	ALT using $y = \frac{-12x^2 + 14x - 5}{x - 1}$ $-24x + 14$	**B1**	
	$\left[\frac{dy}{dx} =\right] \frac{(x-1)(-24x+14)-(-12x^2+14x-5)}{(x-1)^2}$	**M1**	
		A1FT	**FT** on their derivative of 3 term quadratic
(ii)	$\left[\frac{d^2y}{dx^2} =\right] k(x-1)^{-3}$	**M1**	No additional terms
	$k = -6$ isw	**A1**	

(iii)	*their* $\left[\frac{3}{(x-1)^2}-12\right]=0$ and find a value for x	**M1**	$12x^2-24x+9=0$ oe $(2x-3)(2x-1)=0$ oe
	$x = 0.5$ and $x = 1.5$	**A1**	
	$y = 2$ and $y = -22$	**A1**	if **A0 A0** then **A1** for a correct (x, y) pair
	$\frac{-6}{(-0.5)^3}>0$ therefore min when $x = 0.5$ oe	**B1**	or $\left[\frac{-6}{(-0.5)^3}=\right]48$ therefore min when $x = 0.5$ oe
	$\frac{-6}{(0.5)^3}<0$ therefore max when $x = 1.5$ oe	**B1**	or $\left[\frac{-6}{(0.5)^3}=\right]-48$ therefore max when $x = 1.5$ oe **M1A1** is possible from other methods

8.

2 (i)	Clear attempt at quotient rule or equivalent product rule	**M1**	condone omission of brackets
	$\left[\frac{dy}{dx}=\right]\frac{14}{(3-x)^2}$ or $\left[\frac{dy}{dx}=\right]\frac{14}{x^2-6x+9}$ cao or correct simplified equivalent	**A1**	allow recovery from bracketing errors or omissions if implied in correct work to the correct answer
(ii)	$[y = 9]$ $x = 2$	**B1**	
	$\frac{0.07}{\delta x}\approx\left(\textit{their}\left.\frac{dy}{dx}\right\|_{x=2}\right)$ oe	**M1**	condone $\frac{0.07}{\delta x}=\left(\textit{their}\left.\frac{dy}{dx}\right\|_{x=2}\right)$
	0.005 oe	**A1**	not from wrong working; answer only does not score

9.

9 (a)	$\frac{x^2}{2}+x-\frac{1}{x}(+c)$ isw	**B3**	**B1** for each term allow $\frac{x^2}{2}+x+\frac{x^{-1}}{-1}(+c)$ isw for **B3**
(b) (i)	$k\cos(5x+\pi)$ where $k<0$ or $\frac{\cos(5x+\pi)}{5}$	**M1**	
	$\frac{-\cos(5x+\pi)}{5}(+c)$	**A1**	
(ii)	$\frac{-\cos(5(0)+\pi)}{5}-\frac{-\cos(5(-\pi/5)+\pi)}{5}$ or $\frac{-\cos(\pi)}{5}-\left(\frac{-\cos(0)}{5}\right)$	**M1**	correct substitution of the given limits into *their* expression of the form $k\cos(5x+\pi)$, dep on **M1** in (b)(i)
	0.4 oe	**A1**	answer only does not score

10.

5 (i)	$\frac{dy}{dx}=\frac{3}{2(3x+2)}$	**B1**	for correct derivative of log function
	When $x=-\frac{1}{3}$, $y=0$, $\frac{dy}{dx}=\frac{3}{2}$	**B1**	for $y=0$
	Equation of normal: $y=-\frac{2}{3}\left(x+\frac{1}{3}\right)$	**M1** **A1**	**M1** for attempt at a gradient of a perpendicular from differentiation and the equation of the normal
(ii)	$Q\left(0,-\frac{2}{9}\right)$ or $(0,0.22)$ or better	**B1 ft**	Follow through on *their c* from part (i)
	$R\left(0,\frac{1}{2}\ln 2\right)$ or $(0, 0.35)$ or better	**B1**	
	Area of PQR $=\frac{1}{2}\left(\frac{1}{2}\ln 2+\frac{2}{9}\right)\times\frac{1}{3}$ $=0.0948$	**B1**	Allow 0.095

11.

7 (i)	$(0, \sqrt{3})$ or $(0, 1.73)$ or better	**B1**	
(ii)	$\left(\frac{\pi}{6}, 2\right)$ or $(0.524, 2)$ or better	**B1**, **B1**	**B1** for each
(iii)	$\cos\left(x - \frac{\pi}{6}\right) = 0$ $x = \frac{2\pi}{3}$ oe or 2.09 or better	**M1** **A1**	for correct attempt to solve trigonometric equation
(iv)	$2\sin\left(x - \frac{\pi}{6}\right)$ (+c)	**B1**	
(v)	Area $= \left[2\sin\left(x - \frac{\pi}{6}\right)\right]_0^{\frac{2\pi}{3}}$ $= 2 + 1$ $= 3$	**M1** **A1**	for correct use of **their** limits, in radians, into $k\sin\left(x - \frac{\pi}{6}\right)$.

12.

10 (a) (i)	$20U + \frac{1}{2}\left(U + \frac{U}{2}\right)10 = 165$ leading to $U = 6$	**M1** **DM1** **A1**	for realising that area under the graph is needed and attempt to find an area for equating their area to 165 and attempt to solve
(ii)	Gradient of line: -0.3	**M1**, **A1**	**M1** for use of the gradient, must be negative
(b) (i)	27	**B1**	
(ii)	$t^2 = 8\ln 4$ $t = 3.33$ or better	**M1** **A1**	for a correct attempt to solve $e^{\frac{t^2}{8}} = 4$
(iii)	acceleration $= 3\frac{2t}{8}e^{\frac{t^2}{8}}\left(e^{\frac{t^2}{8}} - 4\right)^2$	**M1**, **A1**	**M1** for a correct attempt to differentiate using the chain rule
	When $t = 1$, $a = 6.98$	**M1**, **A1**	**M1** for use of $t = 1$ in their acceleration

13.

6 (i)	$\frac{6x}{3x^2-11}$	**M1** **A1**	**M1** for $\frac{mx}{3x^2-11}$
(ii)	$p=\frac{1}{6}$	**B1**	**FT** for $p=\frac{1}{m}$
(iii)	$\frac{1}{6}\ln\left(3a^2-11\right)-\frac{1}{6}\ln 1=\ln 2$	**M1**	for correct use of limits in $p\ln\left(3x^2-11\right)$ May be implied by following equation
	$\ln\left(3a^2-11\right)=\ln 2^6$	**DM1**	for dealing with logs correctly
	$3a^2-11=64$	**DM1**	for solution of $3a^2-11=k$
	$a=5$ only	**A1**	for 5 obtained from an exact method

14.

10 (i)	$\mathrm{f}\left(x\right)=3x^2-4\mathrm{e}^{2x}\quad(+c)$	**M1** **A1** **A1**	for one correct term for one correct term $3x^2$ or $-4\mathrm{e}^{2x}$ for a second correct term with no extras
	passing through $\left(0,-3\right)$ $-3=3\times 0-4\mathrm{e}^0+c$	**DM1**	for correct method to find *c*.
	$\mathrm{f}\left(x\right)=3x^2-4\mathrm{e}^{2x}+1$	**A1**	for correct equation
(ii)	$\mathrm{f}'\left(0\right)=-8$	**B1**	for $m=\frac{1}{8}$
	Normal: $y+3=\frac{1}{8}x$	**M1**	for equation of normal using $m=\frac{1}{8}$
	$8y+24=x$ $y=2-3x$	**DM1**	for solving normal equation simultaneously with $y=2-3x$ to get a value of x
	leads to $x=\frac{8}{5}$ oe	**A1**	for $x=\frac{8}{5}$, 1.6 oe
	Area $= =\frac{1}{2}\times 3\times\frac{8}{5}=2.4$ oe	**B1**	**FT** for a numerical answer equal to $\left\lvert\frac{1}{2}\times 3\times \text{their } x\right\rvert$

15.

11 (i)	$a=8t-8$ When $t=3$, $a=16$	B1 B1	for $8t-8$ for 16
(ii)	0.5, 1.5	B1,B1	**B1** for each
(iii)	$s=\frac{4}{3}t^3-4t^2+3t$	M1 A1	for at least two terms correct all correct
	when $t=\frac{1}{2}$, $s=\frac{2}{3}$	DM1	for calculating displacement when either $t=\frac{1}{2}$ or $t=\frac{3}{2}$
	when $t=\frac{3}{2}$, $s=0$	DM1	for calculating displacement at $t=\frac{1}{2}$ and doubling.
	total distance travelled = $\frac{4}{3}$	A1	for $\frac{4}{3}$ oe allow 1.33
	Alternative method	M1A1 DM1	As before **DM1** for calculating displacement when $t=0.5$ or for calculating distance travelled between $t=0.5$ and $t=1.5$
		DM1	**DM1** for doubling distance travelled between $t=0.5$ and $t=1.5$ or for adding that distance to displacement at $t=0.5$
		A1	**A1** for $\frac{4}{3}$ oe allow 1.33

16.

5 (i)	$\frac{dy}{dx}=3x^2+4x-7$	B1	
	$x=-2\rightarrow\frac{dy}{dx}=12-8-7=-3$	M1	insert $x=-2$ into *their* gradient and use $(-2,16)$ and *their* gradient of tangent in equation of line.
	Equation of tangent : $\frac{y-16}{x+2}=-3\rightarrow y=-3x+10$	A1	
(ii)	Tangent cuts curve again $x^3+2x^2-7x+2=-3x+10$	M1	equate curve and *their* linear answer from (i).
	$x^3+2x^2-4x-8=0$	A1	
	$(x+2)(x+2)(x-2)=0$	M1	factorise: $(x\pm2)$ and a two or three term quadratic is sufficient. Allow long division
	$x=2,\quad y=4$	A1A1	withhold final **A1** if (2, 4) not clearly identified as their sole answer.

17.

7 **(i)**	$h=\sqrt{9-x^2}$ $A=\frac{\sqrt{9-x^2}}{2}(14+x+x)=\sqrt{9-x^2}\,(7+x)$	**B2/1/0**	Must be clear that $\sqrt{9-x^2}$ is the height of the trapezium. $14+2x$ oe must be seen AG
(ii)	$\frac{dA}{dx}=\sqrt{9-x^2}+(7+x)\frac{1}{2}(9-x^2)^{-0.5}\times -2x$	**M1** **A2/1/0**	product rule on correct function minus 1 each error , allow unsimplified.
	$\frac{dA}{dx}=0\rightarrow 9-x^2=7x+x^2$ $2x^2+7x-9=0$	**M1** **A1**	equate to 0 and simplify to a linear or quadratic equation. correct three term quadratic obtained
	$x=1$ $A=16\sqrt{2}$ or $8\sqrt{8}$ or $\sqrt{512}$ or 22.6	**A1** **A1**	Extra positive answer loses penultimate **A1**. ignore negative solution.

18.

8 **(i)**	$f'(x)=\frac{(x^3+1)9x^2-(3x^3-1)3x^2}{(x^3+1)^2}$	**M1** **A1**	quotient rule or product rule all correct
	$=\frac{12x^2}{(x^3+1)^2}$	**A1**	www beware $9x^6-9x^6$ gets **A0**
(ii)	$\int_1^2\frac{x^2}{(x^3+1)^2}dx=\frac{1}{12}\left[\frac{3x^3-1}{x^3+1}\right]_1^2$	**M1** **A1**	$c\times\frac{3x^3-1}{x^3+1}$ **FT** $c=\frac{1}{their\,12}$
	$=\frac{1}{12}\left[\frac{23}{9}-\frac{2}{2}\right]$	**DM1**	top limit – bottom limit in *their* integral.
	$=\frac{7}{54}$	**A1**	or 0.130 or 0.1296 or 0.12
(iii)	$x=\frac{3y^3-1}{y^3+1}$ $y^3=\frac{x+1}{3-x}$	**B1**	make y^3 or x^3 the subject
	$f^{-1}(x)=\sqrt[3]{\frac{x+1}{3-x}}$	**B1** **B1**	**FT** take cube root (as long as y^3 or x^3 equals a fraction with terms in x or y only) oe **FT** change x and y – can be done at any time
	Domain : $-1\leqslant x\leqslant 2\frac{6}{7}$	**B1**	Allow upper limit of 2.86 . Do not isw

19.

3	**(i)**	$\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right)=\frac{(1+\cos x)\cos x+\sin x\sin x}{(1+\cos x)^2}$	**M1**	Quotient Rule (or Product Rule from $(\sin x)(1+\cos x)^{-1}$)
			A1	correct unsimplified
		$=\frac{\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$	**B1**	use of $\sin^2 x+\cos^2 x=1$ oe
		$=\frac{1+\cos x}{(1+\cos x)^2}$	**A1**	completion
	(ii)	$\int_0^2\left(\frac{1}{1+\cos x}\right)dx=\left[\frac{\sin x}{1+\cos x}\right]_0^2$	**M1**	correct integrand
		awrt 1.56	**A1**	

20.

7	differentiation to obtain answer in the form $p\left(3x^2+8\right)^{\frac{2}{3}}$ or $qx\left(3x^2+8\right)^{\frac{2}{3}}$	**M1**	
	$6x\left(3x^2+8\right)^{\frac{2}{3}}$	**B1**	
	$\frac{dy}{dx}=\frac{5}{3}\times 6x\left(3x^2+8\right)^{\frac{2}{3}}$	**A1**	all correct
	When $\frac{dy}{dx}=0$ only solution is $x=0$	**DM1**	$qx\left(3x^2+8\right)^{\frac{2}{3}}=0$ and attempt to solve
	$x=0$ and $3x^2+8=0$ has no solutions	**A1**	
	Stationary point at $(0,32)$	**A1**	
	correct gradient method with substitution of x values either side of zero or equivalent valid method	**M1**	
	correct conclusion from correct work using a correct $\frac{dy}{dx}$	**A1**	

21.

9(i)	$5+4\left(\sec^2\left(\frac{x}{3}\right)-1\right)$ leading to given answer	**B1**	use of correct identity
9(ii)	$3\tan\left(\frac{x}{3}\right)\quad(+c)$	**B1**	
9(iii)	attempt to integrate using (i) and/or (ii)	**M1**	
	Area $=\int_{\frac{\pi}{2}}^{\pi} 4\sec^2\left(\frac{x}{3}\right)+1\,\mathrm{d}x$	**A1**	all correct
	$\left[12\tan\left(\frac{x}{3}\right)+x\right]_{\frac{\pi}{2}}^{\pi}$	**DM1**	correct method for evaluation using limits in correct order
	$=\left(12\tan\frac{\pi}{3}+\pi\right)-\left(12\tan\frac{\pi}{6}+\frac{\pi}{2}\right)$	**A1**	
	$=8\sqrt{3}+\frac{\pi}{2}$	**A1**	

22.

10(a)	differentiation of a quotient or equivalent product	**M1**	
	correct differentiation of e^{3x}	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\mathrm{e}^{3x}\left(4x^2+1\right)-8x\mathrm{e}^{3x}}{\left(4x^2+1\right)^2}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3\mathrm{e}^{3x}}{4x^2+1}-\frac{8x\mathrm{e}^{3x}}{\left(4x^2+1\right)^2}$	**A1**	everything else correct including brackets where needed, allow unsimplified
10(b)(i)	one term differentiated correctly	**M1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-4\sin\left(x+\frac{\pi}{3}\right)+2\sqrt{3}\cos\left(x+\frac{\pi}{3}\right)$	**A1**	all correct
	When $x=\frac{\pi}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-5$	**A1**	
10(b)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}t}$ $-5\times\frac{\mathrm{d}x}{\mathrm{d}t}=10$ oe	**M1**	correct use of rates of change
	$\frac{\mathrm{d}y}{\mathrm{d}t}=-2$	**A1**	**FT** answer to (i)

23.

2	attempt at differentiating a quotient, must have minus sign and $(x+1)^2$ in the denominator	**M1**	
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{dy}{dx}=\frac{(x+1)\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}-\left(5x^2+4\right)^{\frac{1}{2}}}{(x+1)^2}$	**A1**	all else correct
	When $x=3$, $\frac{dy}{dx}=\frac{11}{112}$	**A1**	must be exact
	Alternative $y=\left(5x^2+4\right)^{\frac{1}{2}}(x+1)^{-1}$	**M1**	attempt to differentiate a product
	for $\left(5x^2+4\right)^{-\frac{1}{2}}$	**B1**	
	for $\frac{1}{2}(10x)\left(5x^2+4\right)^{-\frac{1}{2}}$	**DB1**	
	$\frac{dy}{dx}=\frac{1}{2}10x\left(5x^2+4\right)^{-\frac{1}{2}}(x+1)^{-1}+\left(5x^2+4\right)^{\frac{1}{2}}\left(-(x+1)^{-2}\right)$	**A1**	all else correct
	When $x=3$, $\frac{dy}{dx}=\frac{11}{112}$	**A1**	A1 must be exact

24.

11(i)	me^{2x-1} where m is numeric constant	**M1**	
	$f(x)=\frac{1}{2}e^{2x-1}\quad(+c)$	**A1**	condone omission of $+c$
	$\frac{7}{2}=\frac{1}{2}+c$	**DM1**	correct attempt to find arbitrary constant
	$f(x)=\frac{1}{2}e^{2x-1}+3$	**A1**	must be an equation
11(ii)	ke^{2x-1} where k is a numeric constant	**M1**	
	$f''(x)=2e^{2x-1}$	**A1**	
	$2x-1=\ln\left(\frac{4}{k}\right)$	**DM1**	attempt to equate to 4 and use logarithms
	$x=\frac{1}{2}+\ln\sqrt{2}$	**A1**	

25.

1	Integrates	**M1**	must be clear attempt to integrate at least one term
	$[y=]x^4+x\,(+c)$	**A1**	Both terms correct
	$17=2^4+2+c$	**DM1**	Substitution of $x=2$, $y=17$ to find c
	$y=x^4+x-1$ cao	**A1**	must have $y=$

26.

3(i)	$\frac{2x}{x^2+1}$ final answer	**B2**	**B1** for $\frac{1}{x^2+1}\times(ax+b)$, a or b must be non-zero
3(ii)	$\delta y = their\left(\frac{2(3)}{(3)^2+1}\right)\times h$ or better	**M1**	Substitutes $x=3$ into *their* $\frac{dy}{dx}$ **and** multiplies by h
	$\frac{6}{10}h$ oe	**A1**	

27.

12(i)	$\left[v = \dfrac{\mathrm{d}(3t - \cos 5t + 1)}{\mathrm{d}t} = \right] 3 + 5\sin 5t$	**B2**	**B1** for either with no other terms or for both with 1 extra
	their $(3 + 5\sin 5t) = 0$	**M1**	Must be from an attempt to differentiate
	awrt 0.76	**A1**	0.7570187525
	awrt 1.13	**A1**	1.12793684
	substitutes *their* t values into s (4.07…, 3.58…)	**DM1**	must be two values
	0.48 to 0.49 [m]	**A1**	Final **A1** may imply earlier **A1**s
12(ii)	$25\cos 5t$	**M1**	Differentiating *their* v correctly providing at least 2 terms with one trig function
	–25	**A1**	Ignore +25 following –25

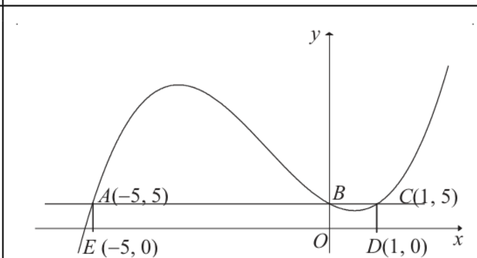
28.

4	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x - 7$ soi	**B1**	
	$m_{\text{normal}} = -\dfrac{1}{5}$ soi	**B1**	finds or uses correct gradient of normal
	$m_{\text{tangent}} = 5$ soi or $(6x-7)\left(-\dfrac{1}{5}\right) = -1$ oe	**M1**	uses $m_1 m_2 = -1$ with numerical gradients
	$6x - 7 = 5$ oe $\Rightarrow x = 2$	**A1**	
	$y = 9$	**A1**	
	$k = 47$	**A1**	
	Alternative method $m_{\text{normal}} = -\dfrac{1}{5}$	**B1**	
	$m_{\text{tangent}} = 5$	**M1**	
	$3x^2 - 12x + 11 - c = 0$ oe	**A1**	
	solving $3x^2 - 12x + 12 = 0$ oe to find $x = 2$	**A1**	
	$y = 9$	**A1**	
	$k = 47$	**A1**	

29.

5(i)	$\left(their\,2x^4\right)(0.2-\ln 5x)+0.4x^5\left(their\,\frac{-5}{5x}\right)$ oe or $their\,0.4x^4-\left(\left(their\,2x^4\right)\ln 5x+0.4x^5\left(their\,\frac{5}{5x}\right)\right)$ oe	**M1**	clearly applies correct form of product rule
	$-2x^4\ln 5x$ isw	**A1**	nfww
5(ii)	$3\ln 5x$ or $\ln 5x+\ln 5x+\ln 5x$	**B1**	
5(iii)	$\frac{3}{-2}\int\left(-2x^4\ln 5x\right)\mathrm{d}x$ oe	**M1**	**FT** $k=2$ from (i) allow for $\frac{3}{2}\int\left(2x^4\ln 5x\right)\mathrm{d}x$ or, when $k=-2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x=-0.2x^5(0.2-\ln 5x)$ or $-\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x=0.4x^5(0.2-\ln 5x)$ oe or, when FT $k=2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x=0.2x^5(0.2-\ln 5x)$ or $\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x=0.4x^5(0.2-\ln 5x)$ oe
	$-\frac{3}{2}\left(0.4x^5(0.2-\ln 5x)\right)[+c]$ oe isw cao	**A1**	nfww; implies M1 An answer of $0.6x^5(0.2-\ln 5x)$ following $k=2$ from (i) implies M1 A0

30.

11(i)	$\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\,[+c]$ isw	**B2**	**B1** for any 3 correct terms
11(ii)	$x^3+4x^2-5x+5=5$ and rearrange to $x\left(x^2+4x-5\right)=0$ oe soi	**B1**	
	Solves *their* $x^2+4x-5[=0]$ soi	**M1**	
	$x=-5,\ x=1$ soi	**A1**	
	$OEAB=25,\ OBCD=5$	**A1**	

11(iii)	Correct or correct **FT** substitution of 0, *their* −5 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{their-5}^{0}$	**M1**	dependent on at least **B1** in (i)
	Correct or correct **FT** substitution of *their* 1, 0 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{0}^{their\,1}$	**M1**	dependent on at least **B1** in (i)
	$their\frac{1175}{12}-their\,OEAB+their\,OBCD-their\frac{49}{12}$ oe	**M1**	for the strategy needed to combine the areas; may be in steps; $97.91\dot{6}-25+5-4.08\dot{3}$
	$\frac{886}{12}$ oe or $73\frac{5}{6}$ oe or $73.8\dot{3}$ rot to 3 or more sig figs	**A1**	all method steps must be seen; not from wrong working If M0 then allow **SC3** for $\int_{-5}^{0}(x^3+4x^2-5x)\,dx-\int_{0}^{1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{-5}^{0}-\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{1}$ $=\left[0-\left(\frac{625}{4}-\frac{500}{3}-\frac{125}{2}\right)\right]-\left[\left(\frac{1}{4}+\frac{4}{3}-\frac{5}{2}\right)-0\right]$ $=\frac{443}{6}$ oe or **SC2** for $\int_{their(-5)}^{0}(x^3+4x^2-5x)\,dx-\int_{0}^{their1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{their(-5)}^{0}-\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{their1}$ $=[F(0)-F(their(-5))]-[F(their1)-F(0)]$

31.

4	correct differentiation of $\ln(3x^2+2)$	**B1**	
	attempt to differentiate a quotient or a product	**M1**	
	$\frac{dy}{dx}=\frac{(x^2+1)\left(\frac{6x}{3x^2+2}\right)-2x\ln(3x^2+2)}{(x^2+1)^2}$	**A1**	all other terms correct.
	When $x=2$, $\frac{dy}{dx}=\frac{5\left(\frac{12}{14}\right)-4\ln 14}{25}$	**M1**	**M1dep** for substitution and attempt to simplify
	$=\frac{6}{35}-\frac{4}{25}\ln 14$	**A2**	**A1** for each correct term, must be in simplest form

32.

9(a)		**B3**	**B1** for line joining $(0,5)$ and $(10,5)$ **B1** for a line joining $(10,0.5)$ and $(30,0.5)$ **B1** all correct with **no solid** line joining $(10,5)$ to $(10,0.5)$
9(b)(i)	3	**B1**	
9(b)(ii)	$\frac{dv}{dt} = -15e^{-5t} + \frac{3}{2}$	**M1**	attempt to differentiate, must be in the form $ae^{-5t} + b$
	When $\frac{dv}{dt} = 0$, $e^{-5t} = 0.1$	**M1**	**M1dep** for equating to zero and attempt to solve, must be of the form $ae^{-5t} = b$, $b > 0$ to obtain an equation in the form $-5t = k$ where k is a logarithm or < 0
	$t = 0.461$	**A1**	
9(b)(iii)	**Either** attempt to integrate, must be in the form $ce^{-5t} + dt^2$	**M1**	
	$s = -\frac{3}{5}e^{-5t} + \frac{3}{4}t^2 \quad (+c)$	**A1**	
	When $t = 0$, $s = 0$ so $c = \frac{3}{5}$	**M1**	**M1dep** for attempt to find c and substitute $t = 0.5$
	$s = 0.738$	**A1**	
	Or attempt to integrate, must be in the form $ce^{-5t} + dt^2$	**M1**	
	$\left[-\frac{3}{5}e^{-5t} + \frac{3}{4}t^2\right]_0^{0.5}$	**A1**	
	correct use of limits	**M1**	**M1dep**
	leading to $s = 0.738$	**A1**	

33.

2	$\frac{dy}{dx} = 10e^{5x} + 3$ an attempt at integration in form $ae^{5x} + bx$	**M1**	
	$y = \frac{10}{5}e^{5x} + 3x \quad (+c)$	**A1**	condone omission of c
	attempt to find c using $x = 0, y = 9$	**M1**	**M1dep**
	$y = 2e^{5x} + 3x + 7$	**A1**	

34.

5(i)	$\frac{5}{14}(7x-10)^{\frac{2}{5}}$	**B2**	**B1** for $k(7x-10)^{\frac{2}{5}}$
5(ii)	$\frac{5}{14}\left[(7x-10)^{\frac{2}{5}}\right]_6^a = \frac{25}{14}$ $\frac{5}{14}(7a-10)^{\frac{2}{5}} - \frac{5}{14}(7\times 6-10)^{\frac{2}{5}} = \frac{25}{14}$ $(7a-10)^{\frac{2}{5}} - 4 = 5$	**M1**	correct application of limits for $k(7x-10)^{\frac{2}{5}}$
	$a = \frac{9^{\frac{5}{2}} + 10}{7}$	**M1**	**M1dep** for evaluation of $(7\times 6-10)^{\frac{2}{5}}$ and correct order of operations to find a, including dealing with power.
	$a = \frac{253}{7}$ or $36\frac{1}{7}$	**A1**	

35.

8(i)	attempt to differentiate a product	M1	
	$\frac{dy}{dx}=\left((x-4)\times\frac{5}{3}\times 3(3x-1)^{\frac{2}{3}}\right)+(3x-1)^{\frac{5}{3}}$	A2	**A1** for $(+)\left((x-4)\times\frac{5}{3}\times 3(3x-1)^{\frac{2}{3}}\right)$ **A1** for $(+)(3x-1)^{\frac{5}{3}}$
	$=(3x-1)^{\frac{2}{3}}((5x-20)+(3x-1))$	M1	use of $(3x-1)^{\frac{5}{3}}=(3x-1)^{\frac{2}{3}}(3x-1)$
	$=(3x-1)^{\frac{2}{3}}(8x-21)$	A1	
8(ii)	When $x=3$, $\frac{dy}{dx}=8^{\frac{2}{3}}\times 3$	M1	$(3\times3-1)^{\frac{2}{3}}\times k$ or $(9-1)^{\frac{2}{3}}\times k$ or $4\times k$ (where k is any number)
	$\partial y=8^{\frac{2}{3}}\times 3\times h$	M1	**M1dep** for *their* $\left((9-1)^{\frac{2}{3}}\times k\right)\times h$
	$\partial y=12h$	A1	

36.

6(i)	$\pi x^2h=500\to h=\frac{500}{\pi x^2}$	B1	Ignore units Condone r for x
6(ii)	$A=2\pi x^2+2\pi xh$	M1	Correct expression for A and insert for *their h*.
	$=2\pi x^2+2\pi x\times\frac{500}{\pi x^2}=2\pi x^2+\frac{1000}{x}$	A1	Answer given Condone r for x.
6(iii)	Differentiate: at least one power reduced by 1	M1	
	$\frac{dA}{dx}=4\pi x-\frac{1000}{x^2}$	A1	
	$\frac{dA}{dx}=0\to x=\sqrt[3]{\frac{1000}{4\pi}}$ isw or $(x=4.3(0))$	A1	
	$A=2\pi(4.3)^2+\frac{1000}{4.3}=349\,\text{cm}^2$	A1	awrt 349
	$\frac{d^2A}{dx^2}=4\pi+\frac{2000}{x^3}$ (>0) or a positive value $(\to$ min$)$	B1	Correct second differential (need not be evaluated) and conclusion. or Examine correct gradient either side of $x=4.3$ and conclusion

37.

7(i)	$v=0 \to \cos 2t = \frac{1}{3}$	M1	set $v=0$ and solve for $\cos 2t$
	$\to t = 0.615$ or 0.616	A1	
7(ii)	$s = \frac{3}{2}\sin 2t - t \quad (+c)$	M1A1	**M1** for $\sin 2t$ and $\pm t$
	$t = \frac{\pi}{4} \to \quad s = 1.5 - \frac{\pi}{4} \quad (= 0.715)$	A1	
7(iii)	$a = -6\sin 2t$	M1A1	**M1** for $-\sin 2t$
	$t = 0.615 \to a = -5.66$ or -5.65 or $-2\sqrt{8}$	A1	condone substitution of degrees

38.

9(i)	$\frac{d}{dx}(\ln x) = \frac{1}{x}$ and $\frac{d}{dx}x^3 = 3x^2$ or $\frac{d}{dx}x^{-3} = -3x^{-4}$	B1	seen
	Substitution of *their* derivatives into quotient rule	M1	
	$\frac{d}{dx}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$ oe	A1	correct completion
9(ii)	$\frac{dy}{dx} = 0 \to 1 - 3\ln x = 0 \qquad \ln x = \frac{1}{3}$	M1	equate given $\frac{dy}{dx}$ to zero and solve for $\ln x$ or x
	$x = e^{\frac{1}{3}}$	A1	seen
	$y = \frac{1}{3e}$	A1	seen
9(iii)	$\frac{\ln x}{x^3} = \int \frac{1 - 3\ln x}{x^4} dx$ oe	M1	use given statement in (i)
	$\int \frac{1}{x^4} dx = \frac{-1}{3x^3}$	B1	seen anywhere
	$\int \frac{\ln x}{x^4} dx = -\frac{1}{9x^3} - \frac{\ln x}{3x^3} \quad (+C)$ oe	A2	**A1** for each term

39.

10(ii)	$\frac{dy}{dx} = \sec^2 x + 6\cos x$	**B2**	**B1** for each term
10(iii)	$\frac{1}{\cos^2 x} + 6\cos x = 7$	**B1**	**B1dep** Replaces $\sec^2 x$ by $\frac{1}{\cos^2 x}$
	$\rightarrow 6\cos^3 x - 7\cos^2 x + 1 = 0$	**B1**	**B1dep** Answer given so all steps must be correct.
10(iv)	$\cos x = 1,\ \frac{1}{2},\ -\frac{1}{3}.$ $\rightarrow x = 0,\ 1.05\left(\text{or } \frac{\pi}{3}\right),\ 1.91$	**A2**	**A1** for 2 values awrt **A1** for third value and no others in range. No credit for answers in degrees

40.

11(i)	$\frac{dy}{dx} = 16 - \frac{54}{x^3}$	**M1**	for $\frac{dy}{dx} = 16 \pm \frac{p}{x^3}$
	Equating to zero and obtaining x^3	**M1**	**M1dep**
	$x = \frac{3}{2},\ y = 36$	**A2**	**A1** for each

11(ii)	**EITHER**: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	$\left(\frac{1}{2}(43+51)\times 2\right)-\int_1^3 16x+\frac{27}{x^2}\,dx$		
	Area of trapezium = $\left(\frac{1}{2}(43+51)\times 2\right)$ oe	**B1**	**FT** from *their P* and *their Q*
	Integration to find area under curve	**M1**	for $\left[px^2+\frac{q}{x}\right]$
	$=\left[8x^2-\frac{27}{x}\right]$	**A1**	Integration correct
	$=\left[8\times 3^2-\frac{27}{3}\right]-\left[8\times 1^2-\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $=94-82$ $=12$	**A1**	
	OR: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	Equation of *PQ*: $y=4x+39$	**B1**	Equation of line **FT** from *their P* and *their Q*
	Integration of their $4x+39-16x-\frac{27}{x^2}$	**M1**	for $\left[px+qx^2+\frac{r}{x}\right]$
	$=\left[39x-6x^2+\frac{27}{x}\right]$	**A1**	All correct
	$=\left[39\times 3-6\times 3^2+\frac{27}{3}\right]$ $-\left[39\times 1-6\times 1^2+\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $=72-60$ $=12$	**A1**	

41.

7(i)	$v=0 \to \cos 2t = \frac{1}{3}$	**M1**	set $v=0$ and solve for $\cos 2t$
	$\to t = 0.615$ or 0.616	**A1**	
7(ii)	$s = \frac{3}{2}\sin 2t - t \quad (+c)$	**M1A1**	**M1** for $\sin 2t$ and $\pm t$
	$t = \frac{\pi}{4} \to \quad s = 1.5 - \frac{\pi}{4} \quad (=0.715)$	**A1**	
7(iii)	$a = -6\sin 2t$	**M1A1**	**M1** for $-\sin 2t$
	$t = 0.615 \to a = -5.66$ or -5.65 or $-2\sqrt{8}$	**A1**	condone substitution of degrees

42.

9(i)	$\frac{d}{dx}(\ln x) = \frac{1}{x}$ and $\frac{d}{dx}x^3 = 3x^2$ or $\frac{d}{dx}x^{-3} = -3x^{-4}$	**B1**	seen
	Substitution of *their* derivatives into quotient rule	**M1**	
	$\frac{d}{dx}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$ oe	**A1**	correct completion
9(ii)	$\frac{dy}{dx} = 0 \to 1 - 3\ln x = 0 \quad \ln x = \frac{1}{3}$	**M1**	equate given $\frac{dy}{dx}$ to zero and solve for $\ln x$ or x
	$x = e^{\frac{1}{3}}$	**A1**	seen
	$y = \frac{1}{3e}$	**A1**	seen
9(iii)	$\frac{\ln x}{x^3} = \int \frac{1-3\ln x}{x^4}dx$ oe	**M1**	use given statement in (i)
	$\int \frac{1}{x^4}dx = \frac{-1}{3x^3}$	**B1**	seen anywhere
	$\int \frac{\ln x}{x^4}dx = -\frac{1}{9x^3} - \frac{\ln x}{3x^3} \quad (+C)$ oe	**A2**	**A1** for each term

43.

11(ii)	**EITHER**: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	$\left(\frac{1}{2}(43+51)\times 2\right)-\int_1^3 16x+\frac{27}{x^2}\,\mathrm{d}x$		
	Area of trapezium = $\left(\frac{1}{2}(43+51)\times 2\right)$ oe	**B1**	**FT** from *their P* and *their Q*
	Integration to find area under curve	**M1**	for $\left[px^2+\frac{q}{x}\right]$
	$=\left[8x^2-\frac{27}{x}\right]$	**A1**	Integration correct
	$=\left[8\times 3^2-\frac{27}{3}\right]-\left[8\times 1^2-\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $=94-82$ $=12$	**A1**	
	OR: When $x=1,\ y=43$ When $x=3,\ y=51$	**B1**	**B1** for both
	Equation of *PQ*: $y=4x+39$	**B1**	Equation of line **FT** from *their P* and *their Q*
	Integration of their $4x+39-16x-\frac{27}{x^2}$	**M1**	for $\left[px+qx^2+\frac{r}{x}\right]$
	$=\left[39x-6x^2+\frac{27}{x}\right]$	**A1**	All correct
	$=\left[39\times 3-6\times 3^2+\frac{27}{3}\right]-\left[39\times 1-6\times 1^2+\frac{27}{1}\right]$	**M1**	**M1dep** for application of limits
	Required area $=72-60$ $=12$	**A1**	

44.

12	$\frac{dy}{dx}=(2x-5)^{\frac{1}{2}}\quad(+c)$	M1	for $k(2x-5)^{\frac{1}{2}}$,
	for $(2x-5)^{\frac{1}{2}}$	A1	
	Substitution to obtain arbitrary constant	M1	**M1 dep** Using $\frac{dy}{dx}=6$ when $x=\frac{9}{2}$
	$\left(\frac{dy}{dx}\right)=(2x-5)^{\frac{1}{2}}+4$	A1	for correct $\frac{dy}{dx}$
	Integration of *their* $k(2x-5)^{\frac{1}{2}}+c$	M1	**M1 dep** on first M1 for integration of $k(2x-5)^{\frac{1}{2}}$ to obtain $m(2x-5)^{\frac{3}{2}}$
	$y=\frac{1}{3}(2x-5)^{\frac{3}{2}}+4x\quad(+d)$	A1	for $\frac{1}{3}(2x-5)^{\frac{3}{2}}+4x$ **FT** *their* (non -zero) constant
	Finding constant	M1	**M1 dep** for obtaining arbitrary constant for $m(2x-5)^{\frac{3}{2}}+nx+d$ using $x=\frac{9}{2},\quad y=\frac{2}{3}$
	$y=\frac{1}{3}(2x-5)^{\frac{3}{2}}+4x-20$	A1	for correct equation

45.

4(i)	$3=\ln(5t+3)$ $e^3=5t+3$ or better	B1	
	$t=3.42$	B1	
4(ii)	$\frac{dx}{dt}=\frac{5}{5t+3}$	M1	for $\frac{k_1}{5t+3}$
	When $t=0$, $\frac{dx}{dt}=\frac{5}{3}$, 1.67 or better	A1	all correct

4(iii)	If $t > 0$ each term in $\frac{k_1}{5t+3} > 0$ so never negative oe	**B1**	**dep** on **M1** in **(ii)** **FT** on *their* $\frac{k_1}{5t+3}$, provided $k_1 > 0$
4(iv)	$\frac{d^2x}{dt^2} = \frac{k_2}{(5t+3)^2}$	**M1**	
	$\frac{d^2x}{dt^2} = -\frac{25}{(5t+3)^2}$ When $t = 0$, $\frac{d^2x}{dt^2} = -\frac{25}{9}$ or -2.78	**A1**	all correct

46.

4(i)	$3 = \ln(5t+3)$ $e^3 = 5t+3$ or better	**B1**	
	$t = 3.42$	**B1**	
4(ii)	$\frac{dx}{dt} = \frac{5}{5t+3}$	**M1**	for $\frac{k_1}{5t+3}$
	When $t = 0$, $\frac{dx}{dt} = \frac{5}{3}$, 1.67 or better	**A1**	all correct
4(iii)	If $t > 0$ each term in $\frac{k_1}{5t+3} > 0$ so never negative oe	**B1**	**dep** on **M1** in **(ii)** **FT** on *their* $\frac{k_1}{5t+3}$, provided $k_1 > 0$
4(iv)	$\frac{d^2x}{dt^2} = \frac{k_2}{(5t+3)^2}$	**M1**	
	$\frac{d^2x}{dt^2} = -\frac{25}{(5t+3)^2}$ When $t = 0$, $\frac{d^2x}{dt^2} = -\frac{25}{9}$ or -2.78	**A1**	all correct

47.

11	When $x=0,\ y=\frac{1}{2}$	B1	For $y=\frac{1}{2}$
	$\frac{dy}{dx}=\frac{1}{2}e^{4x}$	B1	
	$\frac{dy}{dx}=\frac{1}{2}$, Gradient of normal = –2	B1	**FT** on *their* $\frac{dy}{dx}$, must be numeric
	either: Normal $y-\frac{1}{2}=-2x$ or: Gradient of normal = $-\frac{OA}{OB}$	M1	For an attempt at a normal equation passing through *their* $\left(0,\frac{1}{2}\right)$ and a substitution of $y=0$
	When $y=0,\ x=\frac{1}{4}$	A1	
	EITHER: $\int_0^{\frac{1}{4}}\frac{1}{8}e^{4x}+\frac{3}{8}dx$	M1	For attempt to integrate to obtain $k_1e^{4x}+\frac{3}{8}x,\ k_1\neq\frac{1}{8},\ k_1\neq\frac{1}{2}$
	$\left[\frac{1}{32}e^{4x}+\frac{3x}{8}\right]_0^{\frac{1}{4}}$	A1	For correct integration
	Use of limits	M1	**M1dep**
	For area of triangle $=\frac{1}{16}$	B1	**FT** on *their* $x=\frac{1}{4}$
	$=\frac{e}{32}$	A1	final answer in correct form
	OR: $\int_0^{\frac{1}{4}}\frac{1}{8}e^{4x}+\frac{3}{8}-\frac{1}{2}+2x\ dx$	M1	For attempt at subtraction and attempt to integrate to obtain $k_1e^{4x}+\frac{3}{8}x+k_2x+k_3x^2,\ k_1\neq\frac{1}{8}$
	$\left[\frac{1}{32}e^{4x}-\frac{1}{8}x+x^2\right]_0^{\frac{1}{4}}$	A2	–1 for each error for integration
	for use of limits	M1	**M1dep**
	$=\frac{e}{32}$	A1	final answer in correct form

48.

2(i)	$k \times \frac{1}{3x-1}$	**M1**	
	$3 \times \frac{1}{3x-1}$	**A1**	
2(ii)	$x = \frac{11}{15}$ soi	**B1**	
	$0.125 \approx$ *their* $\left.\frac{dy}{dx}\right\|_{x=\textit{their}\frac{11}{15}} \times \delta x$ oe	**M1**	
	0.05 nfww	**A1**	

49.

7(i)	$4\tan x + 4x\sec^2 x$ isw	**B2**	Fully correct **B1** for one correct term as part of e.g. a sum of 2 terms
7(ii)	$\frac{d}{dx}\left(e^{3x+1}\right) = 3e^{3x+1}$	**B1**	
	$\frac{(x^2-1)(\textit{their}3e^{3x+1}) - \textit{their}(2x)e^{3x+1}}{(x^2-1)^2}$	**M1**	
	$\frac{(x^2-1)(3e^{3x+1}) - 2xe^{3x+1}}{(x^2-1)^2}$ oe isw	**A1**	

50.

12(i)	$\tan 30 = \frac{h}{x/2}$ oe	**M1**	
	Correct completion to given answer	**A1**	
	$V = 5\sqrt{3}h^2$ isw	**B1**	

12(ii)(a)	$\frac{dV}{dh} = their 10\sqrt{3}h$ or $\frac{5\sqrt{3}}{2}$	**B1**	**FT** $their V = kh^2$
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ soi	**M1**	
	$\frac{dh}{dt} = \frac{1}{their\left(\frac{dV}{dh}\right)} \times 0.5$	**M1**	
	0.115 or 0.11547 to 0.1155 oe	**A1**	
12(ii)(b)	$\left(\frac{dx}{dt} = \frac{dx}{dh} \times \frac{dh}{dt} =\right) 2\sqrt{3} \times their \frac{1}{5\sqrt{3}}$	**M1**	
	$\frac{2}{5}$	**A1**	