1. 4037/11/M/J/16 Q2

(a) Solve the equation $16^{3x-1} = 8^{x+2}$. [3]

(b) Given that $\dfrac{\left(a^{\frac{1}{3}}b^{-\frac{1}{2}}\right)^3}{a^{-\frac{2}{3}}b^{\frac{1}{2}}} = a^p b^q$, find the value of each of the constants p and q. [2]

2. OMITTED

3. 4037/21/M/J/16 Q3

Do not use a calculator in this question.

(i) Find the value of $-\log_p p^2$. [1]

(ii) Find $\lg\left(\frac{1}{10^n}\right)$. [1]

(iii) Show that $\frac{\lg 20 - \lg 4}{\log_5 10} = (\lg y)^2$, where y is a constant to be found. [2]

(iv) Solve $\log_r 2x + \log_r 3x = \log_r 600$. [2]

4. 4037/22/M/J/16 Q10 (C)

(c) Solve $\log_2(x+1) - \log_2 x = 3$. [3]

5. 4037/12/O/N/16 Q2

By using the substitution $y = \log_3 x$, or otherwise, find the values of x for which

$$3(\log_3 x)^2 + \log_3 x^5 - \log_3 9 = 0 \ .$$ [6]

6. 4037/12/O/N/16 Q11

The variables x and y are such that when $\ln y$ is plotted against x, a straight line graph is obtained. This line passes through the points $x = 4, \ln y = 0.20$ and $x = 12, \ln y = 0.08$.

(i) Given that $y = Ab^x$, find the value of A and of b. [5]

(ii) Find the value of y when $x = 6$. [2]

(iii) Find the value of x when $y = 1.1$. [2]

7. 4037/13/O/N/16 Q5

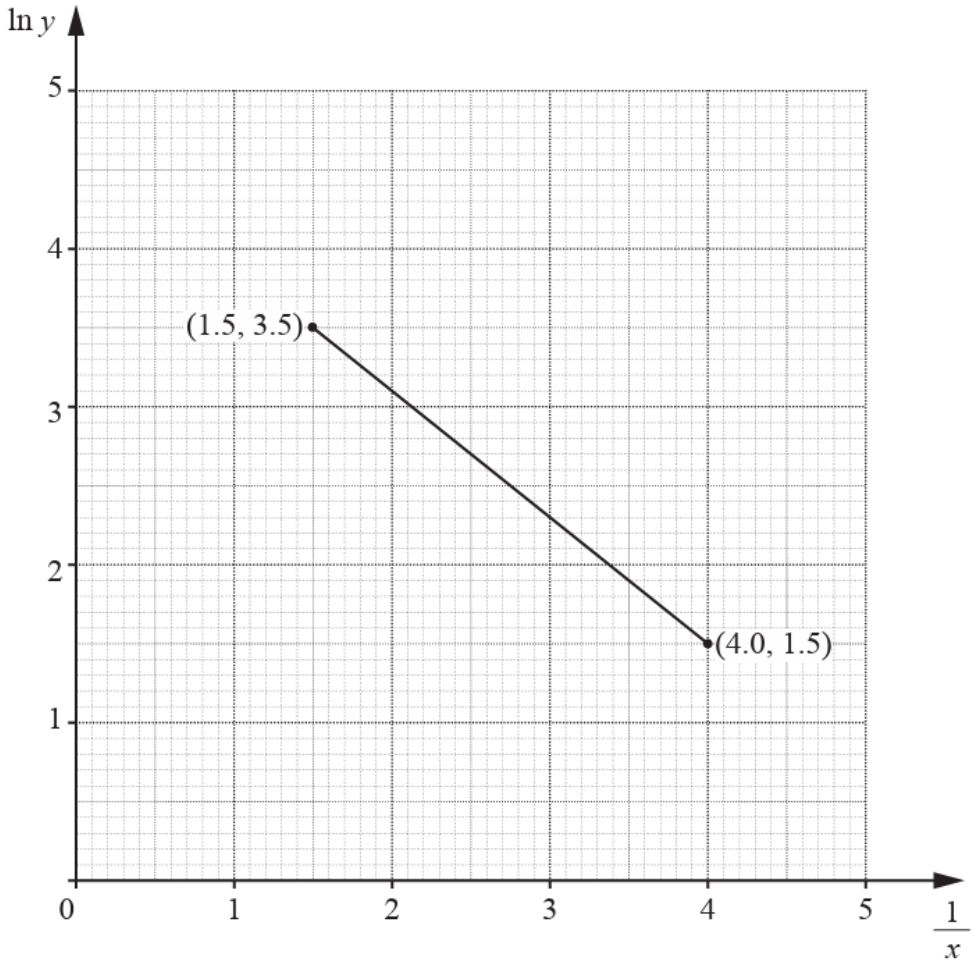
(i) Given that $\log_9 xy = \frac{5}{2}$, show that $\log_3 x + \log_3 y = 5$. [3]

(ii) Hence solve the equations

$$\log_9 xy = \frac{5}{2},$$

$$\log_3 x \times \log_3 y = -6.$$ [5]

8. 4037/13/O/N/16 Q7



The variables x and y are such that when $\ln y$ is plotted against $\frac{1}{x}$ the straight line graph shown above is obtained.

(i) Given that $y = Ae^{\frac{b}{x}}$, find the value of A and of b. [4]

(ii) Find the value of y when $x = 0.32$. [2]

(iii) Find the value of x when $y = 20$. [2]

9. 4037/22/O/N/16 Q3

Solve the equation $2\lg x - \lg\left(\frac{x+10}{2}\right) = 1.$ [5]

10. 4037/22/O/N/16 Q4

The number of bacteria, N, present in a culture can be modelled by the equation $N = 7000 + 2000e^{-0.05t}$, where t is measured in days. Find

(i) the number of bacteria when $t = 10$, [1]

(ii) the value of t when the number of bacteria reaches 7500, [3]

11. 4037/23/O/N/16 Q2

Solve the equation $e^{3x} = 6e^{x}$. [3]

12. 4037/22/M/J/17 Q7

(a) Given that $a^7 = b$, where a and b are positive constants, find,

(i) $\log_a b$, [1]

(ii) $\log_b a$. [1]

(b) Solve the equation $\log_{81} y = -\frac{1}{4}$. [2]

(c) Solve the equation $\frac{32^{x^2-1}}{4^{x^2}} = 16$. [3]

13. 4037/22/O/N/17 Q4

Solve the simultaneous equations

$$\log_2(x+4) = 2\log_2 y,$$

$$\log_2(7y-x) = 4.$$

[5]

14. 4037/23/O/N/17 Q4

Solve the simultaneous equations

$$\log_3(x+1) = 1 + \log_3 y,$$

$$\log_3(x-y) = 2.$$

[5]

15. 4037/11/M/J/18 Q5

The population, P, of a certain bacterium t days after the start of an experiment is modelled by $P = 800e^{kt}$, where k is a constant.

(i) State what the figure 800 represents in this experiment. [1]

(ii) Given that the population is 20 000 two days after the start of the experiment, calculate the value of k. [3]

(iii) Calculate the population three days after the start of the experiment. [2]

16. 4037/11/M/J/18 Q5

(a) Write $(\log_2 p)(\log_3 2) + \log_3 q$ as a single logarithm to base 3. [3]

(b) Given that $(\log_a 5)^2 - 4\log_a 5 + 3 = 0$, find the possible values of a. [3]

17. 4037/12/M/J/18 Q7

A population, B, of a particular bacterium, t hours after measurements began, is given by $B = 1000e^{\frac{t}{4}}$.

(i) Find the value of B when $t = 0$. [1]

(ii) Find the time taken for B to double in size. [3]

(iii) Find the value of B when $t = 8$. [1]

18. 4037/12/M/J/18 Q9

(a) (i) Solve $\lg x = 3$. [1]

(ii) Write $\lg a - 2\lg b + 3$ as a single logarithm. [3]

(b) (i) Solve $x - 5 + \frac{6}{x} = 0$. [2]

(ii) Hence, showing all your working, find the values of a such that $\log_4 a - 5 + 6\log_a 4 = 0$. [3]

19. 4037/13/O/N/18 Q5

(i) Show that $\log_9 4 = \log_3 2$. [2]

(ii) Hence solve $\log_9 4 + \log_3 x = 3$. [3]

20. 4037/22/O/N/18 Q4

Solve

(i) $2^{3x-1} = 6,$ [3]

(ii) $\log_3(y+14) = 1 + \frac{2}{\log_y 3}$. [5]

21. 4037/23/O/N/18 Q6

Solve the simultaneous equations

$$\log_2(x+2y) = 3,$$

$$\log_2 3x - \log_2 y = 1.$$

[5]

22. 4037/11/M/J/19 Q7

(a) Solve $\log_3 x + \log_9 x = 12$. [3]

(b) Solve $\log_4(3y^2-10) = 2\log_4(y-1) + \frac{1}{2}$. [5]

23. 4037/12/M/J/19 Q3.

The number, B, of a certain type of bacteria at time t days can be described by $B = 200e^{2t} + 800e^{-2t}$.

(i) Find the value of B when $t = 0$. [1]

(ii) At the instant when $\frac{dB}{dt} = 1200$, show that $e^{4t} - 3e^{2t} - 4 = 0$. [3]

(iii) Using the substitution $u = e^{2t}$, or otherwise, solve $e^{4t} - 3e^{2t} - 4 = 0$. [2]

24. 4037/22/M/J/19 Q7 a,b

(a) Solve $\lg(x^2 - 3) = 0$. [2]

(b) **(i)** Show that, for $a > 0$, $\dfrac{\ln a^{\sin(2x+5)} + \ln\left(\frac{1}{a}\right)}{\ln a}$ may be written as $\sin(2x+5) + k$, where k is an integer. [3]

25. 4037/12/O/N/19 Q6 (b)

(b) Solve $\log_7 x + 2\log_x 7 = 3$. [4]

26. 4037/13/O/N/19 Q8

(a) Given that $\log_a x = p$ and $\log_a y = q$, find, in terms of p and q,

(i) $\log_a axy^2$, [2]

(ii) $\log_a\left(\frac{x^3}{ay}\right)$, [2]

(iii) $\log_x a + \log_y a$. [1]

(b) Using the substitution $m = 3^x$, or otherwise, solve $3^x - 3^{1+2x} + 4 = 0$. [3]

27. 4037/22/M/J/20

(a) Solve the equation $\dfrac{9^{5x}}{27^{x-2}} = 243.$ [3]

(b) $\log_a \sqrt{b} - \dfrac{1}{2} = \log_b a$, where $a > 0$ and $b > 0$.

Solve this equation for b, giving your answers in terms of a. [5]

28. 4037/22/O/N/20 Q4

Solve the simultaneous equations.

$$\log_3(x+y) = 2$$
$$2\log_3(x+1) = \log_3(y+2)$$

[6]

29. 4037/22/O/N/20 Q10

The number, b, of bacteria in a sample is given by $b = P + Qe^{2t}$, where P and Q are constants and t is time in weeks. Initially there are 500 bacteria which increase to 600 after 1 week.

(a) Find the value of P and of Q. [4]

(b) Find the number of bacteria present after 2 weeks. [1]

(c) Find the first week in which the number of bacteria is greater than 1 000 000. [3]

30. 4037/23/O/N/20 Q8

DO NOT USE A CALCULATOR IN THIS QUESTION.

$$\log_2(y+1) = 3 - 2\log_2 x$$
$$\log_2(x+2) = 2 + \log_2 y$$

(a) Show that $x^3 + 6x^2 - 32 = 0$. [4]

(b) Find the roots of $x^3 + 6x^2 - 32 = 0$. [4]

(c) Give a reason why only one root is a valid solution of the logarithmic equations. Find the value of y corresponding to this root. [2]

31. 4037/12/M/J/21 Q5

(a) Given that $\log_a p + \log_a 5 - \log_a 4 = \log_a 20$, find the value of p. [2]

(b) Solve the equation $3^{2x+1} + 8(3^x) - 3 = 0$. [3]

(c) Solve the equation $4\log_y 2 + \log_2 y = 4$. [3]

32. 4037/14/M/J/21 Q6

(a) Solve the simultaneous equations

$$\log_a(x+y) = 0,$$
$$\log_a(x+1) = 2\log_a y.$$ [4]

(b) Given that $\log_p q^2 \times \log_q p^3 = A$, find the value of the constant A. [3]

33. 4037/21/M/J/21 Q8

In this question, a, b, c and d are positive constants.

(a) (i) It is given that $y = \log_a(x+3) + \log_a(2x-1)$. Explain why x must be greater than $\frac{1}{2}$. [1]

(ii) Find the exact solution of the equation $\dfrac{\log_a 6}{\log_a(y+3)} = 2$. [3]

(b) Write the expression $\log_a 9 + (\log_a b)(\log_{\sqrt{b}} 9a)$ in the form $c + d\log_a 9$, where c and d are integers. [4]

34. 4037/12/O/N/21 Q3

(a) Write $3+2\lg a-4\lg b$ as a single logarithm to base 10. [4]

(b) Solve the equation $3\log_a 4+2\log_4 a=7$. [5]

35. 4037/22/O/N/21 Q5

(a) Solve the following simultaneous equations.

$$e^x + e^y = 5$$
$$2e^x - 3e^y = 8$$

[5]

(b) Solve the equation $e^{(2t-1)} = 5e^{(5t-3)}$. [4]

36. 4037/23/O/N/21 Q4

(a) Solve the equation $\log_6(2x-3)=\frac{1}{2}$. Give your answer in exact form. [2]

(b) Solve the equation $\ln 2u-\ln(u-4)=1$. Give your answer in exact form. [3]

(c) Solve the equation $\frac{3^v}{27^{2v-5}}=9$. [3]

37. 4037/12/M/J/22 Q10

(a) Given the simultaneous equations

$$\lg x + 2\lg y = 1,$$
$$x - 3y^2 = 13,$$

(i) show that $x^2 - 13x - 30 = 0$. [4]

(ii) Solve these simultaneous equations, giving your answers in exact form. [2]

(b) Solve the equation $\log_a x + 3\log_x a = 4$, where a is a positive constant, giving x in terms of a. [5]

38. 4037/21/M/J/22 Q2

(a) Write $2\lg x - \left(\lg(x+6) + \lg 3\right)$ as a single logarithm to base 10. [2]

(b) Hence solve the equation $2\lg x - \left(\lg(x+6) + \lg 3\right) = 0$. [4]

39. 4037/13/O/N/22 Q3b

Solve the equation $\log_3 x + 3 = 10\log_x 3$, giving your answers as powers of 3. [4]

40. 4037/22/O/N/22 Q4

Solve the equation $\log_3(11x-8) = 1 + \frac{2}{\log_x 3}$ given that $x > 1$. [5]

41. 4037/23/O/N/22 Q3

Solve the equation $\lg(2x-1)+\lg(x+2)=2-\lg 4$. [5]