1.

2 **(a)**	$2^{4(3x-1)}=2^{3(x+2)}$ or $4^{2(3x-1)}=4^{\frac{3}{2}(x+2)}$ or $8^{\frac{4}{3}(3x-1)}=8^{x+2}$ or $16^{3x-1}=16^{\frac{3}{4}(x+2)}$	**B1**	**B1** for a correct statement
		M1	for equating indices
	leading to $x=\frac{10}{9}$ cao	**A1**	
(b)	$p=\frac{5}{3}$	**B1**	
	$q=-2$	**B1**	

2. OMITTED

3.

3 **(i)**	-2	**B1**	
(ii)	$-n$	**B1**	
(iii)	$\frac{\lg 5}{\log_5 10}=[(\lg y)^2]$ or $\frac{\lg 20-\lg 4}{1/\lg 5}=[(\lg y)^2]$	**M1**	One log law used correctly
	correct completion to $(\lg 5)^2$ isw	**A1**	answer only does not score
(iv)	$[\log_r]6x^2=[\log_r]600$	**B1**	Condone base missing
	$x=10$ only	**B1**	

4.

(c)	$\frac{x+1}{x}=2^3$ oe www	**M2**	combines logs and anti-logs or **B1** for one correct log move e.g. $\log_2\left(\frac{x+1}{x}\right)=3$ or $\log_2(x+1)-\log_2(x)=\log_2 8$ or $\log_2(x+1)-\log_2(x)=3\log_2 2$
	$x=\frac{1}{7}$ or 0.143 or 0.1428 to 0.1429	**A1**	

5.

3	$3y^2+5y-2=0$	**B1, B1**	**B1** for $5y$ or $5\log_3 x$, **B1** for –2
	$y=\frac{1}{3},\ y=-2$	**M1**	for correct attempt at the solution of *their* quadratic equation
	$x=3^{\frac{1}{3}},\ x=3^{-2}$	**M1**	for dealing with one base 3 logarithm correctly
	$x=1.44,\ x=\frac{1}{9}$	**A1, A1**	**A1** for each

6.

11 (i)	$\ln y=\ln A+x\ln b$	**B1**	may be implied, if equation not seen specifically, by correct values for A and b
	Gradient: $\ln b=-\frac{0.12}{8}$, $=-0.015$	**M1**	for use of gradient to obtain ln b
	$b=0.985$	**A1**	Allow **A1** for $e^{-0.015}$
	Intercept: $\ln A=0.26$	**DM1**	for use of one of the given points correctly
	$A=1.30$	**A1**	Allow **A1** for $e^{0.26}$ or 1.3
	Alternative 1		
	$\ln y=\ln A+x\ln b$	**B1**	
	$0.2=4\ln b+\ln A$	**M1**	for one correct equation
	$0.08=12\ln b+\ln A$	**DM1**	for attempt to obtain either ln$A$ or ln$b$ from simultaneous equations
	$A=1.30$ and $b=0.985$	**A1, A1**	Allow **A1** for b = $e^{-0.015}$ and a = $e^{0.26}$ or 1.3
	Alternative 2		
	$1.22=Ab^4$	**B1**	
	$1.08=Ab^{12}$	**B1**	
		M1	for correct attempt to obtain b or A, must already have **B2**
	$A=1.30$ and $b=0.985$	**A1, A1**	Allow **A1** for b = $e^{-0.015}$ and a = $e^{0.26}$ or 1.3
(ii)	When $x=6$, $\ln y=0.17$	**M1**	for $\ln y=$ *their* $\ln A+6$ *their* $\ln b$ or

7.

5 **(i)**	$\log_9 xy = \log_9 x + \log_9 y$	**M1**	for use of $\log AB = \log A + \log B$
	$= \dfrac{\log_3 x}{\log_3 9} + \dfrac{\log_3 y}{\log_3 9}$	**M1**	for correct method for change of base. Division by $\log_3 9$ should be seen and not implied.
	$= \dfrac{\log_3 x}{2} + \dfrac{\log_3 y}{2} \quad = \dfrac{5}{2}$		
	$\log_3 x + \log_3 y = 5$	**A1**	for dealing with 2 correctly and 'finishing off'
	Alternative method		
	$\log_9 xy = \dfrac{5}{2}$	**M1**	for obtaining xy as a power of 3
	$xy = 9^{\frac{5}{2}} = 3^5$	**M1**	for correct use of $\log_3$
	$\log_3 xy = 5$ $\log_3 x + \log_3 y = 5$	**A1**	for using law for logs and arriving at correct answer
(ii)	$\log_3 x(5 - \log_3 x) = -6$		
	$-(\log_3 x)^2 + 5\log_3 x = -6$	**M1**	for substitution, correct expansion of brackets and manipulation to get a 3 term quadratic
	$(\log_3 x)^2 - 5\log_3 x - 6 = 0$	**A1**	for a correct quadratic equation

8.

7	(i)	$\ln y = \ln A + \frac{b}{x}$	B1	for equation, may be implied, must be using ln unless recovered
		Gradient: $b = -0.8$	B1	for $b = -0.8$ oe
		Intercept or use of equation: $\ln A = 4.7$	B1	for ln A = 4.7 oe, allow 4.65 to 4.75
		$A = 110$	B1	for A = 110, allow 105 to 116 Allow A in terms of e
		Alternative Method $3.5 = \ln A + 1.5b$ and $1.5 = \ln A + 4b$	B1	for one equation
		leading to $b = -0.8$	B1	for $b = -0.8$
		$\ln A = 4.7$	B1	for $\ln A = 4.7$
		and $A = 110$	B1	for $A = 110$ or $e^{4.7}$
		Alternative Method $e^{1.5} = Ae^{4b}$	B1	for $e^{1.5} = Ae^{4b}$ or $4.48 = Ae^{4b}$
		$e^{3.5} = Ae^{1.5b}$	B1	for $e^{3.5} = Ae^{1.5b}$ or $33.1 = Ae^{1.5b}$
		leading to $b = -0.8$	B1	for $b = -0.8$
		and $A = 110$	B1	for $A = 110$ or $e^{4.7}$
	(ii)	When $x = 0.32, \frac{1}{x} = 3.125, \ln y = 2.2$	M1	for a complete method to obtain y, using either the graph, using *their* values in the equation for ln$y$ or using *their* values in the equation for y.
		$y = 9$ (allow 8.5 to 9.5) or $e^{2.2}$	A1	

9.

3	$2\lg x = \lg x^2$	**B1**	soi anywhere
	$1 = \lg 10$	**B1**	soi anywhere
	$\lg x^2 - \lg\left(\frac{x+10}{2}\right) = \lg\left(\frac{2x^2}{x+10}\right)$ oe	**B1**	soi division; logs may be removed
	$2x^2 - 10x - 100 = 0 \rightarrow 2(x+5)(x-10) = 0$	**M1**	obtain correct 3 term quadratic equation and attempt to solve
	$x = 10$ only	**A1**	$x = -5$ must not remain.

10.

4 (i)	$t = 10 \rightarrow N = 7000 + 2000e^{-0.5}$ $= 8213$ or 8210	**B1**	Do not accept non integer responses.
(ii)	$N = 7500 \rightarrow 7500 = 7000 + 2000e^{-0.05t}$	**M1**	insert and make $e^{-0.05t}$ subject
	$e^{-0.05t} = \frac{500}{2000}$		
	$-0.05t = \ln 0.25 \rightarrow t = \frac{\ln 0.25}{-0.05}$	**M1**	take logs and make t the subject
	$= 27.7$ (days)	**A1**	awrt 27.7
(iii)	$\frac{dN}{dt} = -100e^{-0.05t}$	**M1** **A1**	$ke^{-0.05t}$ where k is a constant $k = -100$ or -0.05×2000
	$t = 8 \rightarrow \frac{dN}{dt} = \pm 67$ (.0)	**A1**	awrt ± 67 mark final answer

11.

2	$\ln e^{3x} = \ln 6e^x$		
	$3x = \ln 6e^x$	**M1**	one law of indices/logs
	$3x = \ln 6 + \ln e^x$	**M1**	second law of indices/logs
	$3x = \ln 6 + x$		
	$x = \frac{1}{2}\ln 6$ or $\ln\sqrt{6}$ or 0.896	**A1**	www oe in base 10

12.

7(a)(i)	7	**B1**	
7(a)(ii)	$\frac{1}{7}$ or $\frac{1}{their 7}$	**B1**	**FT** *their* 7 must not be 1 if following through
7(b)	$y=81^{-\frac{1}{4}}$ or $y=3^{-1}$ or $y=9^{-\frac{1}{2}}$ oe	**M1**	Anti-logs
	$y=\frac{1}{3}$ only or 0.333[3….] only	**A1**	nfww; implies the M1; $y=$…. must be seen at least once If M0 then **SC1** for e.g. $81^{-\frac{1}{4}}=\frac{1}{3}$ as final answer

7(c)	$\frac{2^{5(x^2-1)}}{(2^2)^{x^2}}$ oe or $\frac{4^{\frac{5}{2}(x^2-1)}}{4^{x^2}}$ oe or $\frac{32^{x^2}\times 32^{-1}}{4^{x^2}}$ or $\log 32^{x^2-1}-\log 4^{x^2}=\log 16$ oe	**B1**	converts the terms given left hand side to powers of 2 or 4; may have cross-multiplied or separates the power in the numerator correctly or applies a correct log law
	$2^{3x^2-5}=16$ oe $\Rightarrow 3x^2-5=4$ oe or $4^{\frac{3}{2}x^2-\frac{5}{2}}=16$ oe $\Rightarrow \frac{3}{2}x^2-\frac{5}{2}=2$ oe or $\frac{8^{x^2}}{32}=16$ oe $\Rightarrow x^2\log 8=\log 512$ oe or $(x^2-1)\log 32-x^2\log 4=\log 16$ oe	**M1**	combines powers and takes logs or equates powers; or brings down all powers for an equation already in logs condone omission of necessary brackets for M1; condone one slip
	$[x=]\pm\sqrt{3}$ isw cao or ± 1.732050... rot to 3 or more figs. isw	**A1**	

13.

4	$x+4=y^2$	**B1**	
	$7y-x=16$	**B1**	allow 2^4 for16

14.

4	$\log_3 3 = 1$ or $\log_3 9 = 2$	**B1**	implied by one correct equation
	$x + 1 = 3y$	**B1**	
	$x - y = 9$	**B1**	
	solve correct equations for x or y	**M1**	
	$x = 14$ and $y = 5$	**A1**	

15.

5(i)	The number of bacteria at the start of the experiment	**B1**	
5(ii)	$20\,000 = 800e^{kt}$ so $\frac{20\,000}{800} = e^{2k}$ or $\ln 20\,000 = \ln 800 + \ln(e^{2k})$	**M1**	use of given equation and attempt to solve for e^{2k} or use logs correctly
	$2k = \ln 25$	**M1**	correct method to obtain $2k$
	1.61	**A1**	
5(iii)	$P = 800e^{3\ln 5}$	**M1**	Substitution of $t = 3$ in formula using *their k*
	$= 100\,000$	**A1**	answer in range 99800 to 100200

16.

6(a)	$\left(\frac{\log_3 p}{\log_3 2} \times \log_3 2\right) + \log_3 q$	**B1**	

17.

7(i)	1000	**B1**	
7(ii)	$2000 = 1000e^{\frac{t}{4}}$	**B1**	
	$t = 4\ln 2,\quad \ln 16$	**M1**	For $4\ln k$ or $\ln k^4$, $k > 0$
	2.77	**A1**	
7(iii)	$B = 1000e^2$ $= 7389,\ 7390$	**B1**	

18.

9(a)(i)	1000	**B1**	
9(a)(ii)	for use of power rule	**M1**	
	for addition or subtraction rule	**M1**	**dep** on previous **M1**
	$\lg\frac{1000a}{b^2}$	**A1**	Allow $\lg\frac{10^3 a}{b^2}$
9(b)(i)	$x^2 - 5x + 6 = 0$	**M1**	For attempt to obtain a quadratic equation and solve
	$x = 3,\ x = 2$	**A1**	for both
9b(ii)	$(\log_4 a)^2 - 5\log_4 a + 6 = 0$	**M1**	For the connection with **(i)** and attempt to deal with at least one logarithm correctly, either $4^{their 3}$ or $4^{their 2}$
	$a = 64$	**A1**	
	$a = 16$	**A1**	

19.

5(i)	$\log_9 4 = \dfrac{\log_3 4}{\log_3 9}$	**B1**	change of base
	$= \dfrac{1}{2}\log_3 4$ $= \dfrac{1}{2}\log_3 2^2$ or $\log_3 \sqrt{4}$ $= \log_3 2$	**B1**	**Dep** must have B1 for change of base and full working
	Alternative A		
	$\log_9 4 = 2\log_9 2$	**B1**	use of power rule
	$= \dfrac{2\log_3 2}{\log_3 9}$ $= \dfrac{2\log_3 2}{2\log_3 3}$ $= \log_3 2$	**B1**	**Dep** change of base and full working
	Alternative B		
	$x = \log_9 4 \Rightarrow 9^x = 4$ $9^x = 4 \Rightarrow 3^{2x} = 4$	**B1**	correct use of indices to reach $3^{2x} = 4$
	$\Rightarrow 3^x = 2 \Rightarrow x = \log_3 2$ $\therefore \log_9 4 = \log_3 2$	**B1**	**Dep** full working
	Alternative C		
	$\log_9 4 = \dfrac{\log_{10} 4}{\log_{10} 9}$	**B1**	change of base and use of power rule

5(ii)	$\log_3 2 + \log_3 x = 3$ $\log_3 2x = 3$	**B1**	for $\log_3 2x = 3$
	$3^3 = 2x$	**B1**	
	$x = 13.5,\quad x = \dfrac{27}{2}$	**B1**	
	Alternative		
	$\log_3 x = \log_3 27 - \log_3 2$	**B1**	
	$= \log_3 \dfrac{27}{2}$	**B1**	
	$x = 13.5,\quad x = \dfrac{27}{2}$	**B1**	

20.

4(i)	Take logs : $(3x-1)\log 2 = \log 6$	**M1**	
	Make x the subject : $x = \dfrac{\dfrac{\log 6}{\log 2}+1}{3}$ oe	**A1**	
	awrt 1.19 or awrt 1.195	**A1**	
4(ii)	$1 = \log_3 3$	**B1**	
	$\dfrac{2}{\log_y 3} = 2\log_3 y$	**B1**	

21.

6	$\log_2 8 = 3$ or $\log 3x - \log y = \log\frac{3x}{y}$ (any base) or $\log_2 2 = 1$ soi	**B1**	implied by one correct equation
	$x + 2y = 8$	**B1**	
	$\frac{3x}{y} = 2$	**B1**	
	solve correct equations for x or y	**M1**	
	$x = 2$ and $y = 3$	**A1**	

22.

7(a)	<u>Method 1</u> $\log_3 x + \frac{\log_3 x}{\log_3 9} = 12$	**B1**	change to base 3 logarithm
	$\frac{3\log_3 x}{2} = 12$ $x = 3^8$ or $\sqrt[3]{3^{24}}$	**M1**	simplification and dealing with base 3 logarithms to obtain a power of 3
	$x = 6561$	**A1**	

7(a)	<u>Method 2</u> $\frac{\log_9 x}{\log_9 3} + \log_9 x = 12$	**B1**	change to base 9
	$3\log_9 x = 12$ $x = 9^4$ or $\sqrt[3]{9^{12}}$	**M1**	simplification and dealing with base 9 logarithms to obtain a power of 9

7(b)	Method 1 $\log_4(3y^2-10)=\log_4(y-1)^2+\frac{1}{2}$	**B1**	use of power rule
	$\log_4\frac{3y^2-10}{(y-1)^2}=\frac{1}{2}$	**B1**	**DepB1** for use of division rule
	$\frac{3y^2-10}{(y-1)^2}=2$	**B1**	for $\frac{1}{2}=\log_4 2$
	$y^2+4y-12=0$	**M1**	**Dep** on first two B marks simplification to a three term quadratic.
	$y=2$ only	**A1**	

7(b)	Method 2 $\log_4(3y^2-10)=\log_4(y-1)^2+\frac{1}{2}$	**B1**	use of power rule
	$\log_4(3y^2-10)=\log_4(y-1)^2+\log_4 2$	**B1**	for $\log_4 2$
	$3y^2-10=2(y-1)^2$	**B1**	**Dep** on first B1 use of the multiplication rule
	$y^2+4y-12=0$	**M1**	**Dep** on first and third B marks. simplification to a 3 term quadratic
	$y=2$ only	**A1**	

23.

3(i)	1000	**B1**	
3(ii)	$\frac{dB}{dt} = 400e^{2t} - 1600e^{-2t}$	**B1**	
	$3 = e^{2t} - 4e^{-2t}$ oe	**M1**	For equating an equation of the form $ae^{2t} + be^{-2t}$ to 1200 and dividing by 400
	$e^{4t} - 3e^{2t} - 4 = 0$	**A1**	
3(iii)	$(e^{2t}+1)(e^{2t}-4) = 0$	**M1**	For attempt to factorise and solve, dealing with exponential correctly, to obtain $e^{2t} = ...$
	$t = \ln 2,\ \frac{1}{2}\ln 4$ or awrt 0.693 only isw if appropriate	**A1**	

24.

7(a)	$\lg(x^2 - 3) = \lg 1$ soi	**M1**	
	−2 and 2	**A1**	Implies **M1**
7(b)(i)	**Two separate terms in numerator:** $(\sin(2x+5))\ln a$ or $\log_a a^{\sin(2x+5)} = (\sin(2x+5))\log_a a$	**B1**	**Combines terms in numerator:** or $\ln\left(\frac{a^{\sin(2x+5)}}{a}\right)$
	$-\ln a$ or $\log_a a^{-1} = -\log_a a$	**B1**	or $(\sin(2x+5)-1)\ln a$ or $\frac{\ln\left(\frac{a^{\sin(2x+5)}}{a}\right)}{\ln a} = \log_a\left(a^{\sin(2x+5)-1}\right)$
	$\sin(2x+5) - 1$	**B1**	**dep** all previous marks awarded;

25.

6(b)	**Either** $\log_7 x + \frac{2}{\log_7 x} = 3$	**M1**	For change of base.
	$(\log_7 x)^2 - 3\log_7 x + 2 = 0$ $\log_7 x = 1, \quad \log_7 x = 2$	**M1**	Dep for forming a 3 term quadratic equation in $\log_7 x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 7 logarithm correctly
		A1	For both
	Or $\frac{1}{\log_x 7} + 2\log_x 7 = 3$	**M1**	For change of base
	$2(\log_x 7)^2 - 3\log_x 7 + 1 = 0$ $\log_x 7 = 1, \quad \log_x 7 = 0.5$	**M1**	Dep for forming a 3 term quadratic equation in $\log_x 7$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base x logarithm correctly
		A1	For both
	Or $\frac{\lg x}{\lg 7} + 2\frac{\lg 7}{\lg x} = 3$ or $\lg 1000$	**M1**	For change of base
	$(\lg x)^2 - 3\lg 7(\lg x) + 2(\lg 7)^2 = 0$ $\lg x = 2\lg 7 \qquad \lg x = \lg 7$	**M1**	Dep for forming a 3 term quadratic equation in $\lg x$ and a correct attempt to solve
	$x = 7, \quad x = 49$	**M1**	Dep on both previous M marks for dealing with a base 10 logarithm correctly
		A1	For both, must be exact

26.

8(a)(i)	$\log_a a + 2\log_a y + \log_a x$	**M1**	for $\log_a a + \log_a x + \log_a y^2$ **and** $\log_a y^2 = 2\log_a y$
	$1 + 2q + p$	**A1**	
8(a)(ii)	$3\log_a x - \log_a y - \log_a a$	**M1**	for $\log_a x^3 - (\log_a a + \log_a y)$ **and** $\log_a x^3 = 3\log_a x$
	$3p - q - 1$ or $3p - (q+1)$	**A1**	
8(a)(iii)	$\frac{1}{p} + \frac{1}{q}$	**B1**	
8(b)	$m - 3m^2 + 4 = 0$	**M1**	for obtaining a quadratic in m or 3^x
	$m = \frac{4}{3}, \ (-1)$ $x = \frac{\lg\frac{4}{3}}{\lg 3}, \ x = \frac{\ln\frac{4}{3}}{\ln 3}$ or $\lg_3 \frac{4}{3}$	**M1**	Dep for attempt to solve quadratic and deal with 3^x correctly
	$x = 0.262$ only	**A1**	

27.

9(a)	$\frac{3^{10x}}{3^{3x-6}} [= 243]$ oe or $\log 9^{5x} - \log 27^{x-2} = \log 243$ oe	**B1**	
	$3^{7x+6} = 3^5$ soi oe or $5x(\log 9) - (x-2)\log 27 = \log 243$	**M1**	
	$x = -\frac{1}{7}$	**A1**	

9(b)	$\frac{1}{2}\log_a b - \frac{1}{2} = \frac{1}{\log_a b}$ or $\frac{\frac{1}{2}}{\log_b a} - \frac{1}{2} = \log_b a$	**B2**	**B1** for bringing down the power of $\frac{1}{2}$ e.g. $\frac{1}{2}\log_a b$ or for a change of base e.g. $\frac{1}{\log_a b}$
	Clears the fraction and rearranges $\frac{1}{2}(\log_a b)^2 - \frac{1}{2}\log_a b = 1$ oe $(\log_a b)^2 - \log_a b - 2 = 0$ oe or let $x = \log_a b$ $\quad x^2 - x - 2 = 0$ oe or $\frac{1}{2} - \frac{1}{2}\log_b a = (\log_b a)^2$ $0 = 2(\log_b a)^2 + \log_b a - 1$ oe or let $y = \log_b a$ $\quad 2y^2 + y - 1 = 0$	**M1**	
	$(\log_a b - 2)(\log_a b + 1)$ oe or $(2\log_b a - 1)(\log_b a + 1)$	**M1**	
	[$\log_a b = 2$, $\quad \log_a b = -1$ or $\log_b a = \frac{1}{2}$, $\quad \log_b a = -1$ leading to] $b = a^2$, $b =$ oe	**A1**	

28.

4	$x + y = 9$	**B1**	
	$(x+1)^2 = y + 2$	**B1**	
	$x + (x+1)^2 - 2 = 9$ or $(10 - y)^2 = y + 2$	**M1**	Replace y or x. Allow unsimplified using *their* three term expressions both containing x and y terms. Condone one sign or arithmetic error. Result must be a quadratic function.
	$x^2 + 3x - 10\ (= 0)$ or $y^2 - 21y + 98\ (= 0)$	**A1**	Correct 3 term quadratic
	$x = -5$ and $x = 2$ or $y = 7$ and $y = 14$ or $(x+5)(x-2)$ or $(y-7)(y-14)$	**M1**	**Dep** on correct method to solve their quadratic
	$x = 2$ and $y = 7$ **only**	**A1**	Reject $x = -5$, $y = 14$ as log -4 is not appropriate

29.

10(a)	$P + Q = 500$ and $P + Qe^2 = 600$	**B1**	
	$Q = \dfrac{100}{(e^2 - 1)} = 15.7$ or 15.6	**2**	**M1** for attempt to solve by removing P from two equations both containing 3 terms **A1** awrt
	$P = 484$ or 485	**A1**	awrt
10(b)	$B = 484.3 + 15.65e^4 = 1338$	**B1**	Integer value rounded down from 1338… if seen.
10(c)	$e^{2t} = \dfrac{1000000 - 484.3}{15.65}$	**M1**	Make e^{2t} the subject
	$2t = \ln\left(\dfrac{1000000 - 484.3}{15.65}\right)$	**M1**	Take logs correctly where $e^{2t} > 0$ or $e^{n} > 0$
	$[t = 5.5(3) \text{ or } t = 5.5\ldots] \rightarrow 6^{\text{th}}$ week.	**A1**	nfww

30.

8(a)	$x^2(y+1)=8$ oe	**B1**	
	$x+2=4y$ oe	**B1**	
	$x^2\left(\frac{x+2}{4}+1\right)=8$	**M1**	eliminate y from correct equations
	$x^3+6x^2-32=0$	**A1**	**answer given**
8(b)	$x=2$ or $x=-4$ seen or $(x-2)$ or $(x+4)$ seen	**B1**	
	find quadratic factor	**M1**	x^2 and 16 or long division to x^2+kx or x^2 and –8 or long division to x^2+kx not from expanding two linear factors
	$(x^2+8x+16)$ or (x^2+2x-8)	**A1**	
	$(x-2)(x+4)^2$ and $x=2,-4,-4$	**A1**	answer only without working earns B1 above only
8(c)	no real value for $\log_2(-4)$ or $\log_2(-4+2)$	**B1**	must identify specific term in one of original equations and use $x=-4$
	$y=1$	**B1**	

31.

5(a)	$p=16$	**2**	**B1** for $\log_a\frac{5p}{4}=\log_a 20$ oe **B1** for 16, nfww
5(b)	$(3(3^x)-1)(3^x+3)=0$	**M1**	For recognition of a correct quadratic in 3^x and attempt to factorise or use quadratic formula
	$3^x=\frac{1}{3}$ $x=-1$	**2**	**M1 dep** for a correct attempt to solve $3^x=k,\ k>0$ **A1** for one solution only, must be from a correct solution.

5(c)	$\log_y 2 = \dfrac{1}{\log_2 y}$ or $\log_2 y = \dfrac{1}{\log_y 2}$ or $\log_y 2 = \dfrac{\log_a 2}{\log_a y}$ **and** $\log_2 y = \dfrac{\log_a y}{\log_a 2}$	**B1**	May be implied
	$4(\log_y 2)^2 - 4(\log_y 2) + 1 = 0$ $(2\log_y 2 - 1)^2 = 0$, $\log_y 2 = \dfrac{1}{2}$ or $(\log_2 y)^2 - 4(\log_2 y) + 4 = 0$ $(\log_2 y - 2)^2 = 0$, $\log_2 y = 2$ or $(\log_a y)^2 - 4(\log_a 2)(\log_a 4)\log_a y$ $+4(\log_a 2)^2 = 0$ $(\log_a y - 2\log_a 2)^2 = 0$ $\log_a y = 2\log_a 2$	**M1**	For obtaining a 3 term quadratic equation in either $\log_y 2$ or $\log_2 y$ and solving to obtain $\log_y 2 = k$ or $\log_2 y = k$, may be implied or equivalent using an alternative base.
	$y = 4$	**A1**	nfww

32.

6(a)	$x + y = 1$	**B1**	
	$x + 1 = y^2$	**B1**	
	leading to $y^2 + y - 2 = 0$ or $x^2 - 3x = 0$ $y = 1$, $x = 0$ only	**2**	**M1** for obtaining a 3-term quadratic equation in y or a 2-term quadratic equation in x. **A1** for the one pair only
6(b)	$2\log_p q \times \dfrac{3}{\log_p q} = 6$ or $\dfrac{2}{\log_q p} \times 3\log_q p = 6$	**3**	**M1** for use of power rule to obtain either $3\log_q p$ or $2\log_p q$. **M1** for change of base of logarithm for 1 term. **A1** for 6 nfww.

33.

8(a)(i)	Valid explanation e.g. This ensures the argument of both logarithms is greater than 0	**B1**	
8(a)(ii)	$\log_a 6 = \log_a (y+3)^2$ oe	**B1**	
	$(y+3)^2 = 6$	**M1**	
	$y = -3 + \sqrt{6}$ oe only final answer	**A1**	
8(b)	Within a complete expression: Correct change of base to *a*: $\log_{\sqrt{b}} 9a = \frac{\log_a 9a}{\log_a \sqrt{b}}$ Correct use of power law: $\log_a \sqrt{b} = \frac{1}{2}\log_a b$ Correct use of addition/multiplication law: $\log_a 9a = \log_a 9 + \log_a a$ Correct use of $\log_a a = 1$	**M3**	**M2** for 2 or 3 correct steps within complete expression or **M1** for 1 correct step within complete expression
	leading to $2 + 3\log_a 9$. nfww	**A1**	

36.

4(a)	$2x-3=6^{\frac{1}{2}}$ oe, soi	**M1**	
	$x=\dfrac{6^{\frac{1}{2}}+3}{2}$ or $x=\dfrac{\sqrt{6}+3}{2}$	**A1**	
4(b)	$\ln\dfrac{2u}{u-4}=\ln e$ soi or $\ln\dfrac{2u}{u-4}=1$ soi or $\ln 2u=\ln e(u-4)$ soi	**M1**	Condone one sign or bracketing error
	$\dfrac{2u}{u-4}=e$ or $2u=e(u-4)$ oe	**M1**	**FT** *their* logarithmic equation
	$u=\dfrac{4e}{e-2}$ or $u=\dfrac{-4e}{2-e}$ or equivalent exact form	**A1**	
4(c)	$\dfrac{3^v}{\left(3^3\right)^{2v-5}}=3^2$ oe soi or $\dfrac{9^{\frac{v}{2}}}{\left(9^{\frac{3}{2}}\right)^{2v-5}}=9$ oe soi or $\log 3^v-\log 27^{2v-5}=\log 9$ oe soi	**B1**	
	$15-5v=2$ oe or $v\log 3-(2v-5)\log 27=\log 9$	**M1**	**FT** *their* exponential equation in the same base or *their* logarithmic equation with any consistent base, providing *their* exponential or logarithmic equation has at most one sign or arithmetic error
	$v=\dfrac{13}{5}$ oe	**A1**	

37.

10(a)(i)	xy^2 soi	**B1**	Simplification of the left-hand side of the first equation
	$1 = \lg 10$ soi	**B1**	Simplification of right-hand side of equation
	$x - 3\left(\frac{10}{x}\right) = 13$	**M1**	For substitution of y^2 into linear equation oe and attempt to simplify
	$x^2 - 13x - 30 = 0$	**A1**	**AG** – must see sufficient detail to justify the given answer
	Alternative $y^2 = \frac{(x-13)}{3}$	**(B1)**	
	$\lg x + \lg\frac{(x-13)}{3} = 1$	**(M1)**	For attempt at substitution in the log equation
	$\frac{x(x-13)}{3} = 10$ oe	**(B1)**	
	$x^2 - 13x - 30 = 0$	**(A1)**	**AG** – must see sufficient detail to justify the given answer
10(a)(ii)	$x = 15$ only	**B1**	
	$y = \sqrt{\frac{2}{3}}$ or $\frac{\sqrt{6}}{3}$ or exact equivalent only	**B1**	isw once exact value seen
10(b)	$\log_a x + \frac{3}{\log_a x}$ or $\frac{1}{\log_x a} + 3\log_x a$	**B1**	For an appropriate change of base
	$(\log_a x)^2 - 4\log_a x + 3 = 0$ or $3(\log_x a)^2 - 4\log_x a + 1 = 0$	**M1**	For an attempt to obtain a 3-term quadratic equation in terms of $\log_a x$ or $\log_x a$, equated to zero.
	$\log_a x = 3 \quad \log_a x = 1$ or $\log_x a = \frac{1}{3}, \quad \log_x a = 1$	**M1**	**Dep** on previous **M** mark for correct solution of *their* quadratic equation
	$x = a$	**A1**	Must be from completely correct work
	$x = a^3$	**A1**	Must be from completely correct work

38.

2(a)	$\lg\frac{x^2}{3(x+6)}$ oe, nfww	**B2**	**B1** for any two log laws applied correctly e.g. $\lg\frac{x^2}{x+6}-\lg 3$
2(b)	$\lg\frac{x^2}{3(x+6)}=\lg 1$ or $10^0=\frac{x^2}{3(x+6)}$	**B1**	FT *their* $\lg\frac{x^2}{3(x+6)}$ providing a single logarithm
	$x^2-3x-18=0$	**B1**	**dep** on **B2** in **part (a)**
	Factorises or solves their 3-term quadratic	**M1**	
	$x = 6$ indicated as only solution	**A1**	**dep** on **B2** in **part (a)**

39.

3(b)	$[\log_3 x+3=]\frac{10}{\log_3 x}$ oe or $\frac{1}{\log_x 3}[+3=10\log_x 3]$ oe	**B1**	For change of base
	$(\log_3 x)^2+3\log_3 x-10=0$ or $10(\log_x 3)^2-3\log_x 3-1=0$ $\log_3 x=-5$ $\log_3 x=2$ or $\log_x 3=-\frac{1}{5}$ $\log_x 3=\frac{1}{2}$	**M1**	**Dep** on previous **B** mark, for attempt to obtain a 3-term quadratic equation and attempt to solve to obtain 2 solutions of the form $\log_3 x=p$ or $\log_x 3=q$

40.

4	$\log_3\left(\frac{11x-8}{x^2}\right)=1$ or $\log_3(11x-8)=\log_3(3x^2)$ soi OR $\log_x\left(\frac{11x-8}{3}\right)=2$ or $\log_x(11x-8)=\log_x(3x^2)$ soi	**M2**	**M1** for correct use change of base in a correct equation so that all logs have consistent base: $\log_x 3=\frac{\log_3 3}{\log_3 x}$ or $\log_x 3=\frac{1}{\log_3 x}$ oe, soi **OR** $\log_3(11x-8)=\frac{\log_x(11x-8)}{\log_x 3}$ oe, soi
	$3x^2-11x+8[=0]$ oe, nfww	**A1**	
	$(3x-8)(x-1)$ [= 0]	**M1**	**FT** *their* 3-term quadratic **dep** on at least **M1** previously awarded
	$x=\frac{8}{3}$ or 2.67 or 2.666[6…] rot to 3 or more dp as only solution	**A1**	

41.

3	$\lg((2x-1)(x+2))=\lg\frac{100}{4}$ oe or $10^2=4(2x-1)(x+2)$ oe	**M2**	**M1** for one log law correctly applied within a correct equation e.g. $\lg 4(2x-1)(x+2)\quad=2$
	$2x^2+3x-27[=0]$	**A1**	Collects terms to correct 3-term quadratic in solvable form
	$(2x+9)(x-3)[=0]$	**M1**	**dep** on at least **M1** previously awarded Factorises *their* $2x^2+3x-27$ or solves *their* $2x^2+3x-27=0$
	$x=3$ indicated as only valid solution	**A1**	nfww