1. 4037/11/M/J/22 Q3

A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

(a) Write down the range of f. [1]

A function g is such that $g(x) = 5x - 7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

(c) Find the solution of the equation $f'(x) = g^{-1}(x)$. [6]

2. 4037/12/O/N/22 Q8

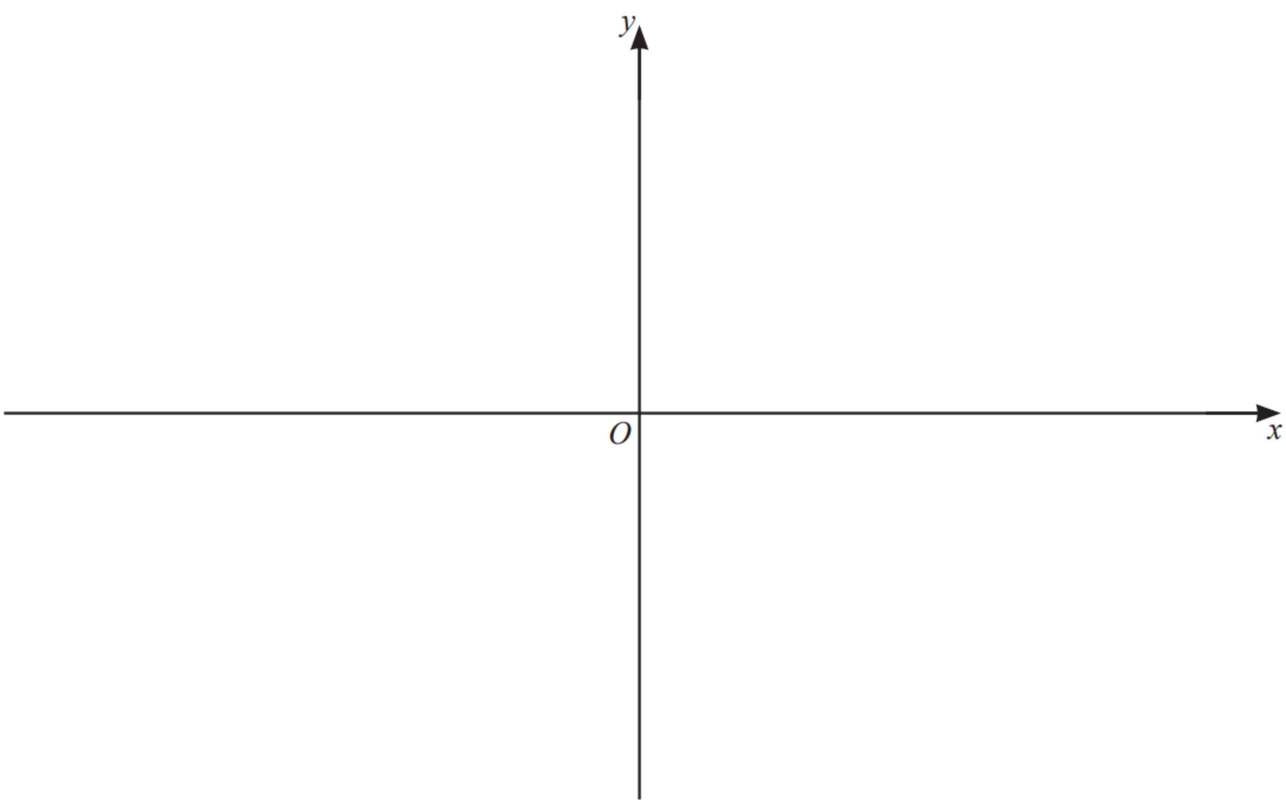
A function $f(x)$ is such that $f(x) = \ln(2x+3) + \ln 4$, for $x > a$, where a is a constant.

(a) Write down the least possible value of a. [1]

(b) Using your value of a, write down the range of f. [1]

(c) Using your value of a, find $f^{-1}(x)$, stating its range. [4]

each graph with the coordinate axes. Label each of your graphs. [4]



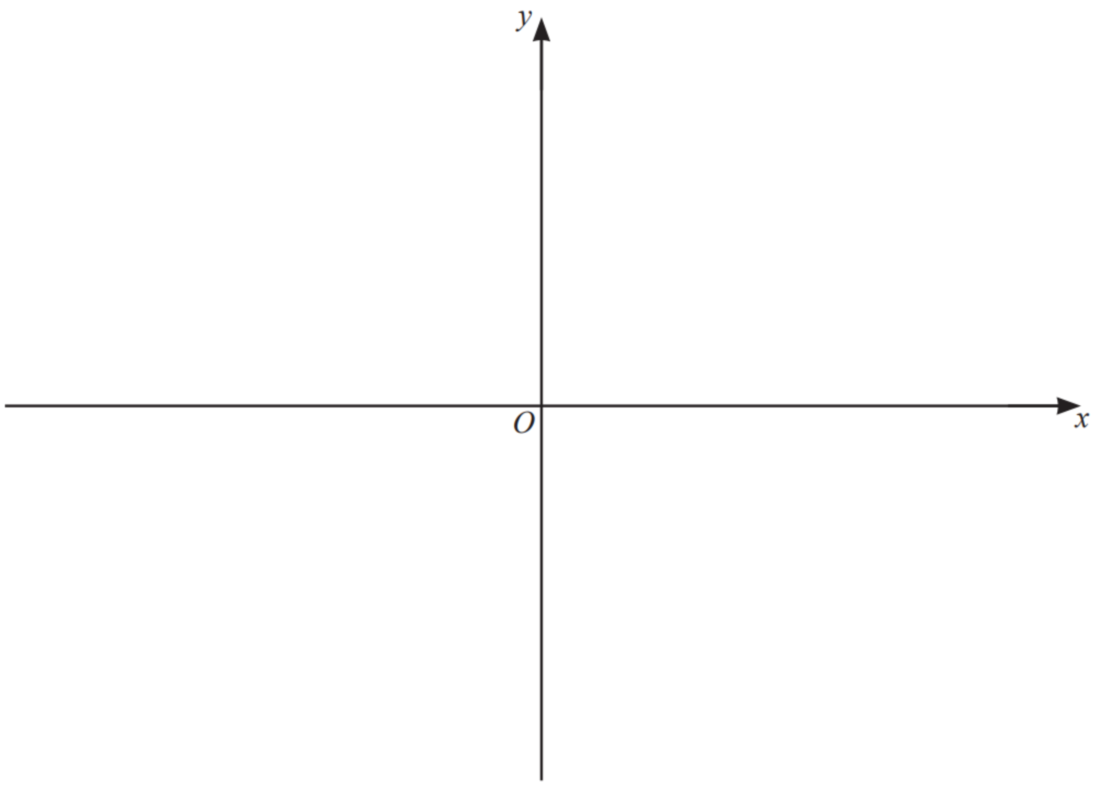
3. 4037/13/O/N/22 Q6

A function $f(x)$ is such that $f(x) = e^{3x} - 4$, for $x \in \mathbb{R}$.

(a) Find the range of f. [1]

(b) Find an expression for $f^{-1}(x)$. [2]

(c) On the axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ stating the exact values of the intercepts with the coordinate axes. [4]



4. 4037/11/M/J/21 Q5

The functions f and g are defined as follows.

$$f(x) = x^2 + 4x \quad \text{for } x \in \mathbb{R}$$

$$g(x) = 1 + e^{2x} \quad \text{for } x \in \mathbb{R}$$

(a) Find the range of f. [2]

(b) Write down the range of g. [1]

(c) Find the exact solution of the equation $fg(x) = 21$, giving your answer as a single logarithm. [4]

5. 4037/22/M/J/21 Q13

The functions f and g are defined, for $x > 0$, by

$$\mathrm{f}(x) = \frac{2x^2 - 1}{3x},$$

$$\mathrm{g}(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for $\mathrm{fg}(x)$. [2]

(b) **(i)** Given that f^{-1} exists, write down the range of f^{-1}. [1]

(ii) Show that $\mathrm{f}^{-1}(x) = \dfrac{px + \sqrt{qx^2 + r}}{4}$, where p, q and r are integers. [4]

6. 4037/24/M/J/21 Q9

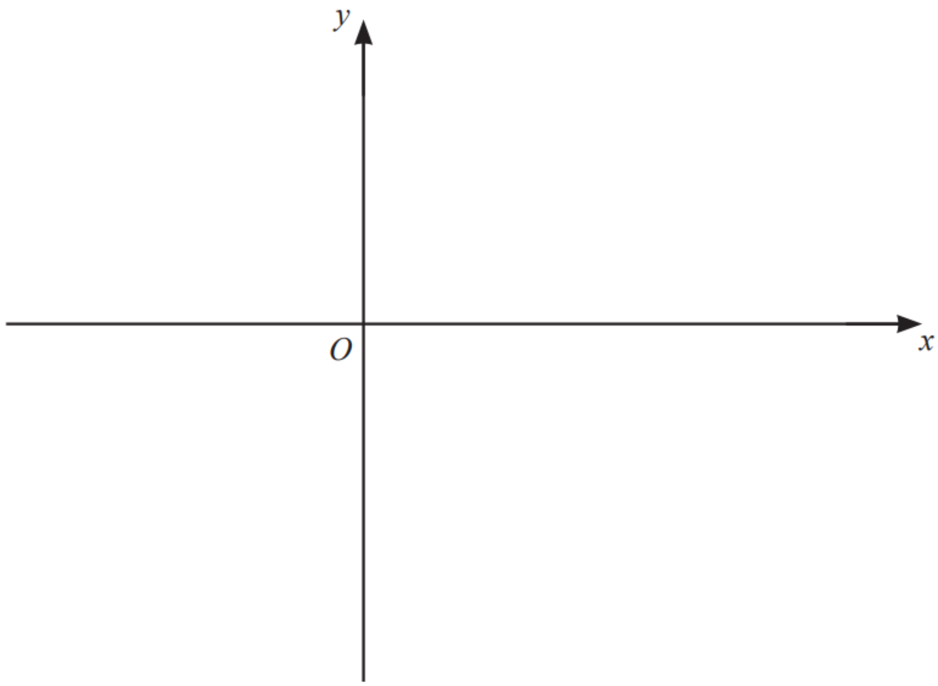
A function f is defined, for all real values of x, by $f(x) = 3 + e^{5x}$.

(a) Find the range of f. [1]

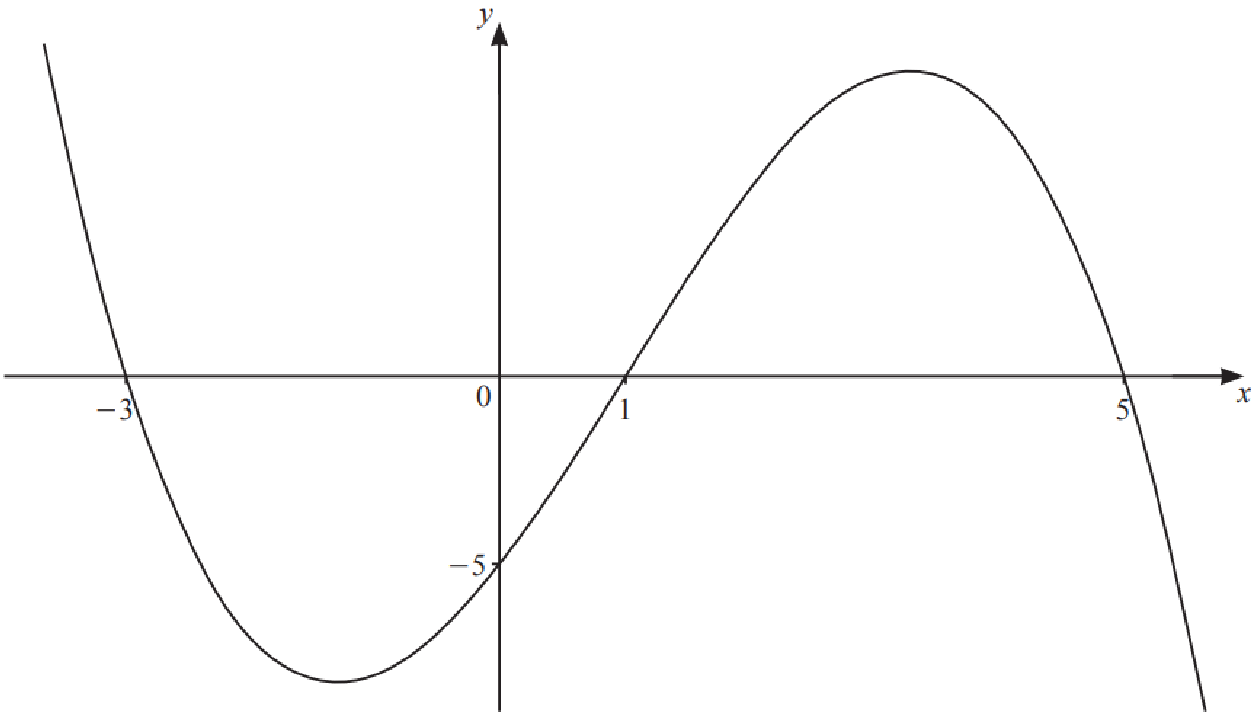
(b) Find an expression for $f^{-1}(x)$ and state its domain. [3]

(c) Solve $f^{-1}(x) = 0$. [2]

(d) Sketch the graph of $y = f(x)$. Hence, on the same axes, sketch the graph of $y = f^{-1}(x)$. Give the coordinates of any points where the graphs cross the axes. [4]



7. 4037/12/O/N/21 Q1



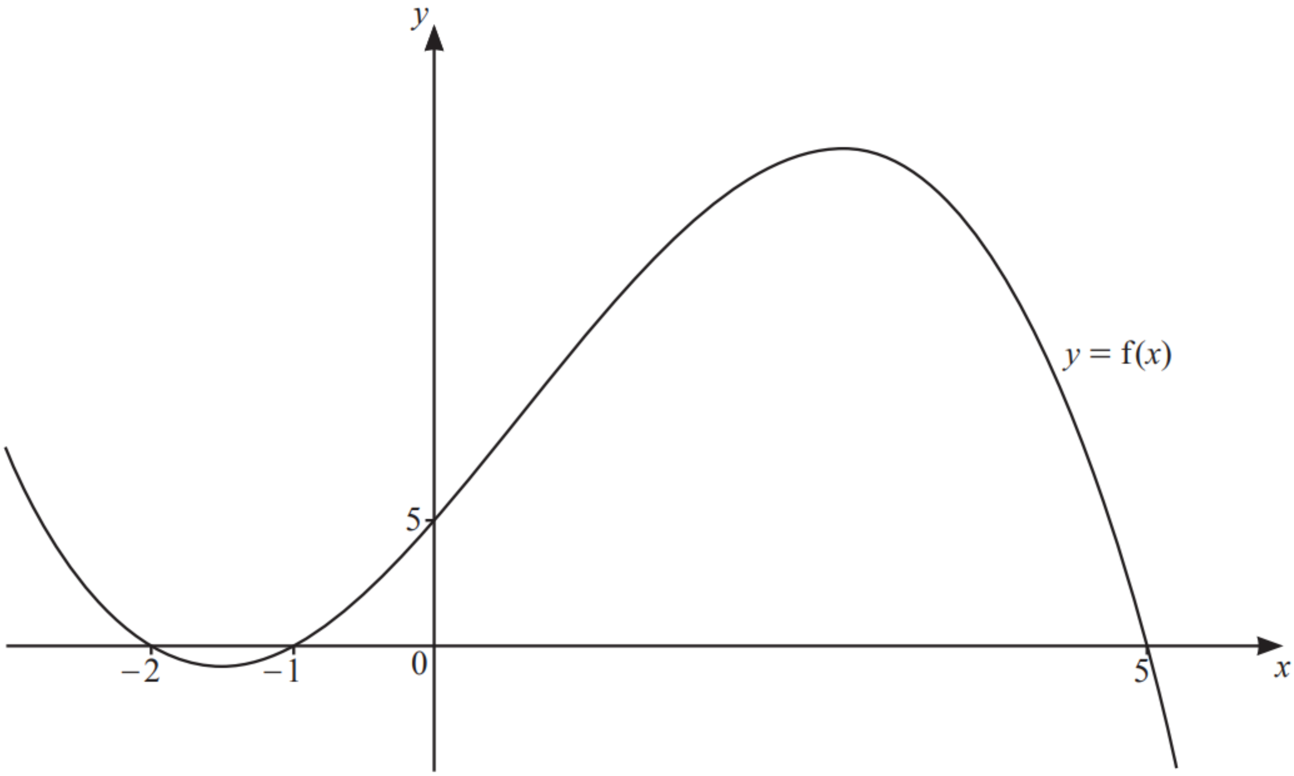
The diagram shows the graph of the cubic function $y = f(x)$. The intercepts of the curve with the axes are all integers.

(a) Find the set of values of x for which $f(x) < 0$. [1]

(b) Find an expression for $f(x)$. [3]

8. 4037/11/M/J/20 Q1

The diagram shows the graph of a cubic curve $y = \mathrm{f}(x)$.



(a) Find an expression for $\mathrm{f}(x)$. [2]

(b) Solve $\mathrm{f}(x) \leqslant 0$. [2]

9. 4037/12/M/J/20 Q5

$$f : x \mapsto (2x+3)^2 \quad \text{for } x > 0$$

(a) Find the range of f. [1]

(b) Explain why f has an inverse. [1]

(c) Find f^{-1}. [3]

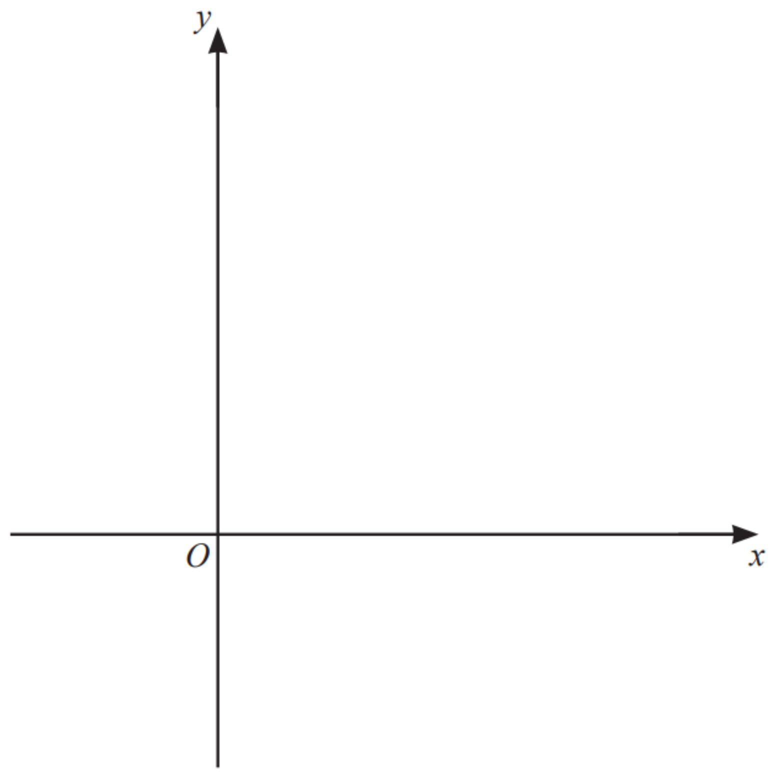
(d) State the domain of f^{-1}. [1]

(e) Given that $g : x \mapsto \ln(x+4)$ for $x > 0$, find the exact solution of $fg(x) = 49$. [3]

10. 4037/21/M/J/20 Q11

The function f is defined by $f(x) = \ln(2x+1)$ for $x \geqslant 0$.

(a) Sketch the graph of $y = f(x)$ and hence sketch the graph of $y = f^{-1}(x)$ on the axes below. [3]



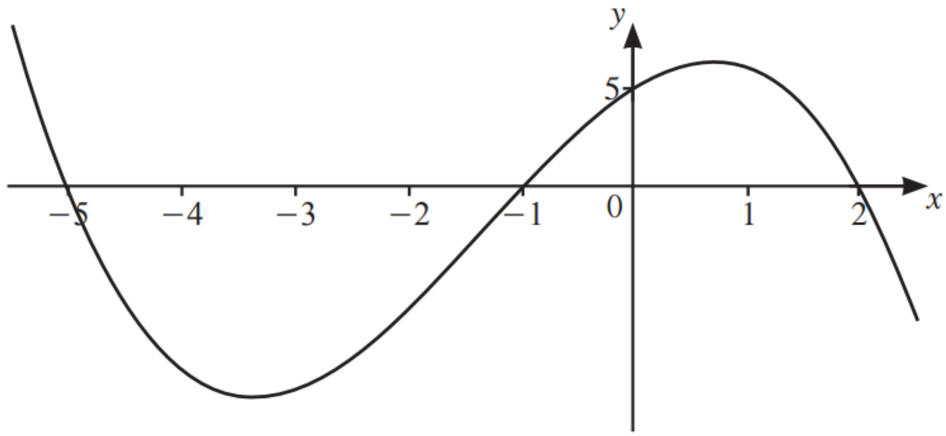
The function g is defined by $g(x) = (x-4)^2 + 1$ for $x \leqslant 4$.

(b) **(i)** Find an expression for $g^{-1}(x)$ and state its domain and range. [4]

(ii) Find and simplify an expression for $\text{fg}(x)$. [2]

(iii) Explain why the function gf does not exist. [1]

11. 4037/12/O/N/20 Q2



The diagram shows the graph of $y = f(x)$, where $f(x)$ is a cubic polynomial.

(a) Find $f(x)$. [3]

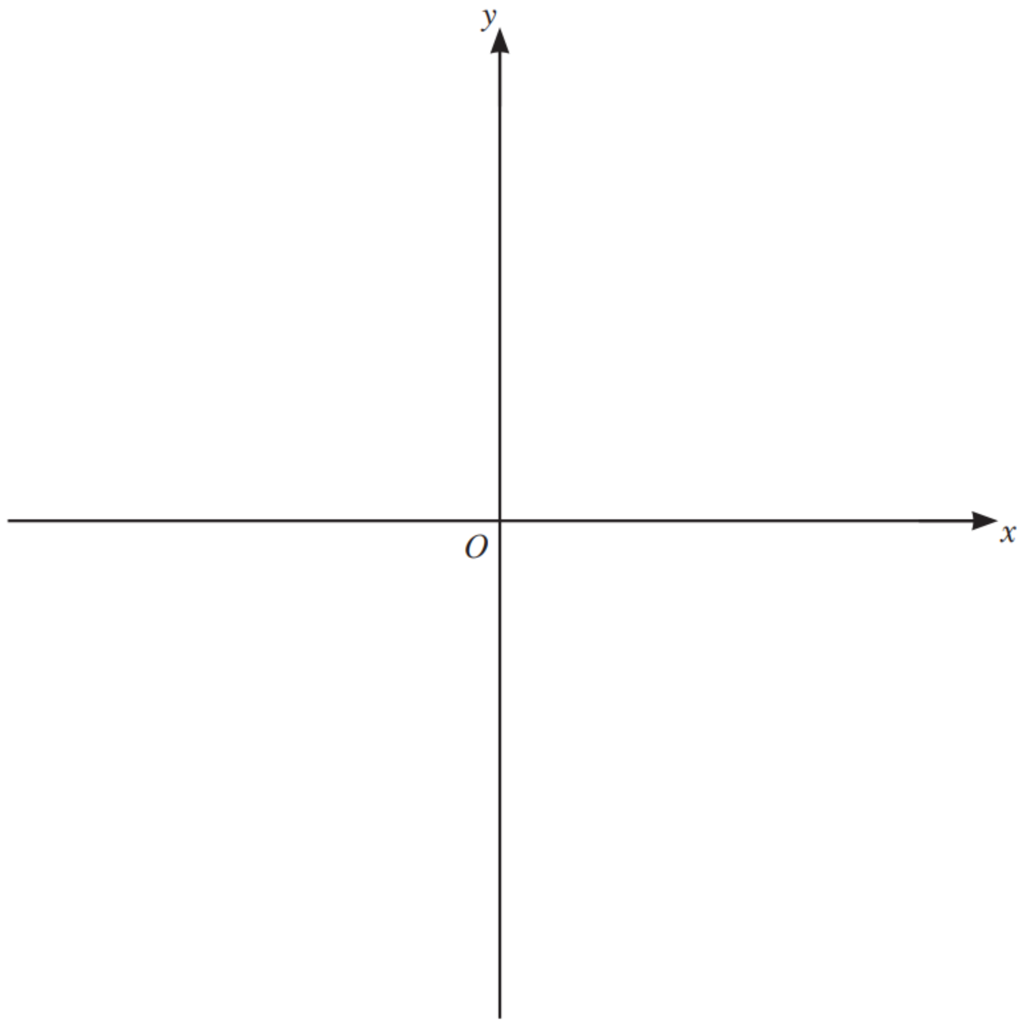
(b) Write down the values of x such that $f(x) < 0$. [2]

12. 4037/12/O/N/20 Q6

$$f(x) = x^2 + 2x - 3 \quad \text{for } x \geqslant -1$$

(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $f(x)$ has an inverse. [1]

(b) On the axes below, sketch the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.



[4]

13. 4037/13/O/N/20 Q3

(a) $$f(x) = 4\ln(2x - 1)$$

(i) Write down the largest possible domain for the function f. [1]

(ii) Find $f^{-1}(x)$ and its domain. [3]

(b) $$g(x) = x + 5 \quad \text{for } x \in \mathbb{R}$$

$$h(x) = \sqrt{2x - 3} \quad \text{for } x \geqslant \frac{3}{2}$$

Solve $gh(x) = 7$. [3]

14. 4037/11/M/J/19 Q8

It is given that $f(x) = 5e^x - 1$ for $x \in \mathbb{R}$.

(i) Write down the range of f. [1]

(ii) Find f^{-1} and state its domain. [3]

It is given also that $g(x) = x^2 + 4$ for $x \in \mathbb{R}$.

(iii) Find the value of fg(1). [2]

(iv) Find the exact solutions of $g^2(x) = 40$. [3]

15. 4037/22/M/J/19 Q12

(a) The functions f and g are defined by

$$f(x) = 5x - 2 \quad \text{for } x > 1,$$
$$g(x) = 4x^2 - 9 \quad \text{for } x > 0.$$

(i) State the range of g. [1]

(ii) Find the domain of gf. [1]

(iii) Showing all your working, find the exact solutions of $gf(x) = 4$. [3]

16. 4037/12/O/N/19 Q5

(a) It is given that $f : x \mapsto \sqrt{x}$ for $x \geqslant 0$,
$g : x \mapsto x + 5$ for $x \geqslant 0$.

Identify each of the following functions with one of f^{-1}, g^{-1}, fg, gf, f^2, g^2.

(i) $\sqrt{x+5}$ [1]

(ii) $x - 5$ [1]

(iii) x^2 [1]

(iv) $x + 10$ [1]

(b) It is given that $h(x) = a + \frac{b}{x^2}$ where a and b are constants.

(i) Why is $-2 \leqslant x \leqslant 2$ not a suitable domain for $h(x)$? [1]

(ii) Given that $h(1) = 4$ and $h'(1) = 16$, find the value of a and of b. [2]

17. 4037/23/O/N/19 Q10

The functions f and g are defined by

$$f(x) = \ln(3x+2) \quad \text{for } x > -\frac{2}{3},$$
$$g(x) = e^{2x} - 4 \quad \text{for } x \in \mathbb{R}.$$

(i) Solve $gf(x) = 5$. [5]

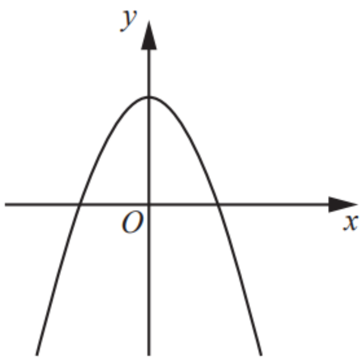
(ii) Find $f^{-1}(x)$. [2]

(iii) Solve $f^{-1}(x) = g(x)$. [4]

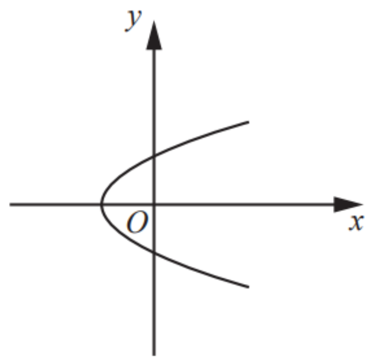
18. 4037/11/M/J/18 Q3

Diagrams **A** to **D** show four different graphs. In each case the whole graph is shown and the scales on the two axes are the same.

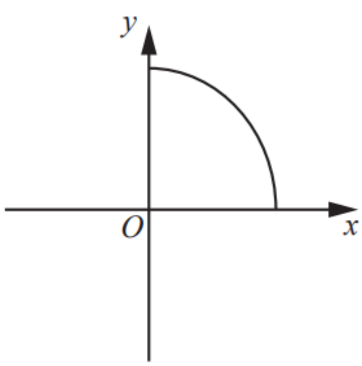
A



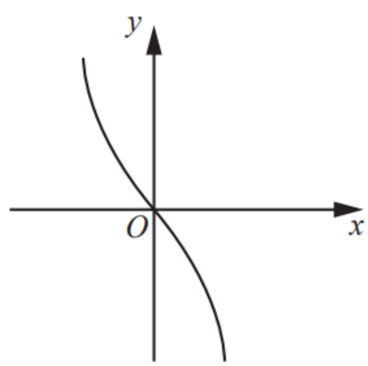
B



C



D



Place ticks in the boxes in the table to indicate which descriptions, if any, apply to each graph. There may be more than one tick in any row or column of the table. [4]

	A	**B**	**C**	**D**
Not a function				
One-one function				
A function that is its own inverse				
A function with no inverse				

19. 4037/21/M/J/18 Q5

The function f is defined by $\mathrm{f}(x) = \dfrac{1}{2x - 5}$ for $x > 2.5$.

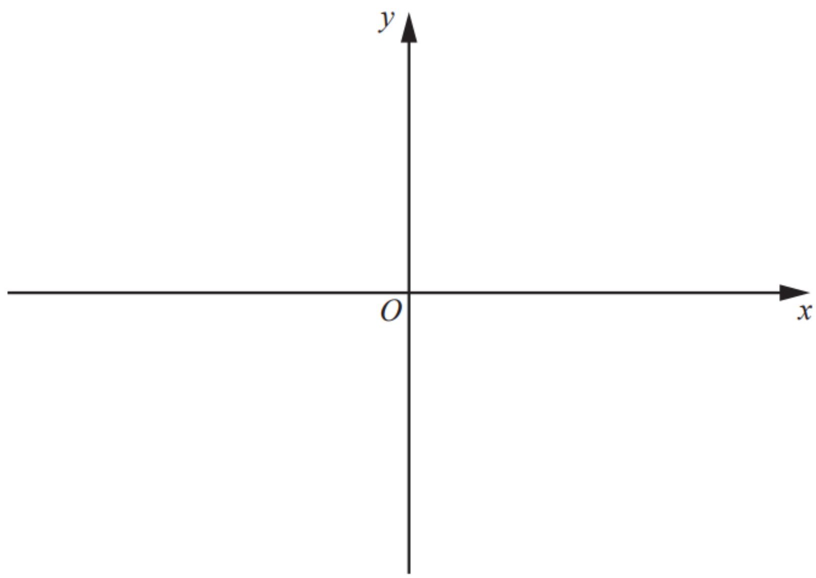
(i) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(ii) State the domain of $\mathrm{f}^{-1}(x)$. [1]

(iii) Find an expression for $\mathrm{f}^{2}(x)$, giving your answer in the form $\dfrac{ax + b}{cx + d}$, where a, b, c and d are integers to be found. [3]

20. 4037/22/M/J/18 Q10

(a) **(i)** On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

(b) The functions g and h are defined by

$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

(i) Find $hg(x)$. [1]

(ii) Find $(hg)^{-1}(x)$. [2]

(c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

21. 4037/13/O/N/18 Q8

$$f(x) = 5 + \sin\frac{x}{4} \quad \text{for} \quad 0 \leqslant x \leqslant 2\pi \text{ radians}$$

$$g(x) = x - \frac{\pi}{3} \quad \text{for} \quad x \in \mathbb{R}$$

(i) Write down the range of $f(x)$. [2]

(ii) Find $f^{-1}(x)$ and write down its range. [3]

(iii) Solve $2fg(x) = 11$. [4]

22. 4037/22/O/N/18 Q11

The functions f and g are defined for real values of $x \geqslant 1$ by

$$f(x) = 4x - 3,$$

$$g(x) = \frac{2x+1}{3x-1}.$$

(i) Find $gf(x)$. [2]

(ii) Find $g^{-1}(x)$. [3]

(iii) Solve $fg(x) = x - 1$. [4]

23. 4037/11/M/J/17 Q4

(a) It is given that $f(x) = 3e^{-4x} + 5$ for $x \in \mathbb{R}$.

(i) State the range of f. [1]

(ii) Find f^{-1} and state its domain. [4]

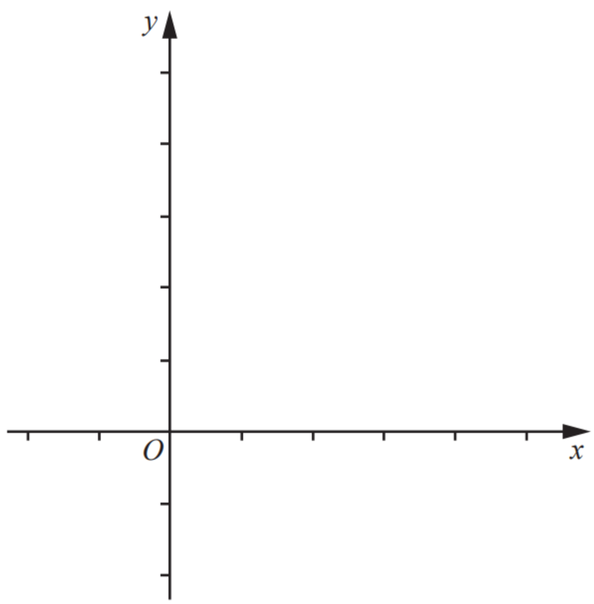
(b) It is given that $g(x) = x^2 + 5$ and $h(x) = \ln x$ for $x > 0$. Solve $hg(x) = 2$. [3]

24. 4037/22/M/J/17 Q9

A function f is defined, for $x \leqslant \frac{3}{2}$, by $f(x) = 2x^2 - 6x + 5$.

(i) Express $f(x)$ in the form $a(x-b)^2 + c$, where a, b and c are constants. [3]

(ii) On the same axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the geometrical relationship between them. [3]



(iii) Using your answer from part **(i)**, find an expression for $f^{-1}(x)$, stating its domain. [3]

25. 4037/22/M/J/17 Q12

The function g is defined, for $x > -\frac{1}{2}$, by $g(x) = \frac{3}{2x+1}$.

(i) Show that $g'(x)$ is always negative. [2]

(ii) Write down the range of g. [1]

The function h is defined, for all real x, by $h(x) = kx + 3$, where k is a constant.

(iii) Find an expression for $hg(x)$. [1]

(iv) Given that $hg(0) = 5$, find the value of k. [2]

(v) State the domain of hg. [1]

26. 4037/12/O/N/17 Q6

Functions f and g are defined, for $x > 0$, by

$$f(x) = \ln x,$$
$$g(x) = 2x^2 + 3.$$

(i) Write down the range of f. [1]

(ii) Write down the range of g. [1]

(iii) Find the exact value of $f^{-1}g(4)$. [2]

(iv) Find $g^{-1}(x)$ and state its domain. [3]

27. 4037/23/O/N/17 Q6

The functions f and g are defined for real values of x by

$$f(x) = (x+2)^2 + 1,$$

$$g(x) = \frac{x-2}{2x-1},\ x \neq \frac{1}{2}.$$

(i) Find $f^2(-3)$. [2]

(ii) Show that $g^{-1}(x) = g(x)$. [3]

(iii) Solve $gf(x) = \frac{8}{19}$. [4]