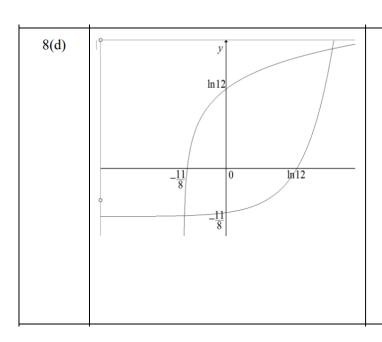
3(a)	$f(x) \in \mathbb{R}$ oe	B1	Must be using correct notation, allow $y \in$
3(b)	$5(\ln(3x+1)) - 7 = 13$	M1	For correct order
	$x = \frac{e^4 - 1}{2}$	2	M1 for a correct attempt to solve to get x =, allow one sign error Dep on previous M mark A1 all correct must be exact
3(c)	$(\mathbf{f}'(x) =) \ \frac{2}{2x+1}$	2	M1 for $\frac{a}{2x+1}$ A1 all correct
	$(g^{-1}(x) =) \frac{x+7}{5}$	B1	soi
	$2x^2 + 15x - 3 = 0$	M1	for equating and forming a 3-term quadratic equation = 0
	x = 0.195, -7.69	M1	For solution of <i>their</i> 3-term quadratic
	x = 0.195	A1	For discounting negative root.

8(a)	-1.5	B1	
8(b)	$f\in\mathbb{R}$	B1	Allow $y \in \mathbb{R}$, \mathbb{R} , $-\infty < f(x) < \infty$ oe, $f(x) \in \mathbb{R}$
8(c)	ln(8x+12) or $ln(4(2x+3))$	B1	May be implied
	$f^{-1}(x) = \frac{e^x - 12}{8}$ oe	2	M1 for attempt to find the inverse, allow one sign error A1 allow $y =$
	Range: $f^{-1} > their(-1.5)$	В1	Must be correct notation, follow through on <i>their</i> (a) $f^{-1}(x) > their(-1.5)$, $y > their(-1.5)$
	Alternative		
	$f^{-1}(x) = \frac{e^{x-\ln 4} - 3}{2}$ oe	(3)	B1 for $e^{x-\ln 4}$ or $e^{y-\ln 4}$ M1 for attempt to find the inverse, allow one sign error A1 allow $y = \dots$
	Range: $f^{-1} > their(-1.5)$	(B1)	Must be correct notation, follow through on <i>their</i> (a) $f^{-1}(x) > their(-1.5)$, $y > their(-1.5)$
•	8(b)	8(b) $f \in \mathbb{R}$ 8(c) $\ln(8x+12) \text{ or } \ln(4(2x+3))$ $f^{-1}(x) = \frac{e^x - 12}{8} \text{ oe}$ Range: $f^{-1} > their(-1.5)$ Alternative $f^{-1}(x) = \frac{e^{x-\ln 4} - 3}{2} \text{ oe}$	8(b) $f \in \mathbb{R}$ B1 8(c) $\ln(8x+12) \text{ or } \ln(4(2x+3))$ B1 $f^{-1}(x) = \frac{e^x - 12}{8} \text{ oe}$ Range: $f^{-1} > their(-1.5)$ B1 Alternative $f^{-1}(x) = \frac{e^{x-\ln 4} - 3}{2} \text{ oe}$ (3)



- **B1** for correct shape of f(x) in quadrants 1, 2 and 3, with asymptotic behaviour
- **B1** for $\ln 12$ and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape.
- **B1** for correct shape of $f^{-1}(x)$ in quadrants 1, 3 and 4, with asymptotic behaviour
- **B1** for ln12 and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape and intersect at least once with y = f(x)

6(a)	f>-4	B1	Allow $y > -4$ or $-4 < f < \infty$ or $f \in (-4, \infty)$
6(b)	$\left[\mathbf{f}^{-1}(x)\right] = \frac{1}{3}\ln(x+4)$	2	M1 for a correct method to find the inverse, allow one sign error Must be in the form of $3x = \ln(y \pm 4)$ or $3y = \ln(x \pm 4)$ A1 allow $y =$
6(c)	1 n4 x	4	B1 for $f(x)$ with correct shape in quadrant 1, 3 and 4 and appropriate asymptotic behaviour B1 for -3 on the y-axis and $\frac{1}{3}\ln 4$ on the x-axis for $f(x)$ must have the correct shape B1 for $f^{-1}(x)$ with correct shape in quadrant 1, 2 and 3 and appropriate asymptotic behaviour B1 for -3 on the x-axis and $\frac{1}{3}\ln 4$ on the y-axis for $f^{-1}(x)$ must have correct shape and intersect at least once

5(a)	f ≥ -4	2	M1 for a valid method to find the least value of $x^2 + 4x$ A1 for $f \ge -4$, $y \ge -4$ or $f(x) \ge -4$
5(b)	g > 1	B1	Allow $y > 1$ or $g(x) > 1$
5(c)	$(1+e^{2x})^2 + 4(1+e^{2x})[=21]$	M1	
	$e^{4x} + 6e^{2x} - 16 = 0$ $(e^{2x} + 8)(e^{2x} - 2) = 0$	M1	Dep for quadratic in terms of e^{2x} and attempt to solve to obtain $e^{2x} = k$
	$e^{2x} = 2$ $x = \frac{1}{2} \ln k$	M1	Dep on both previous M marks, for attempt to solve $e^{2x} = k$
	$x = \ln \sqrt{2} \text{ or } \ln 2^{\frac{1}{2}}$	A1	

13(a)	$[fg(x) =] \frac{2\left(\frac{1}{x}\right)^2 - 1}{3\left(\frac{1}{x}\right)} \text{oe}$	M1	
	$[fg(x) =] \frac{2 - x^2}{3x}$ or $\frac{2}{3x} - \frac{x}{3}$	A1	mark final answer
13(b)(i)	$f^{-1} \ge 0$	B1	
13(b)(ii)	$2x^{2} - 3xy - 1 = 0$ or $2y^{2} - 3xy - 1 = 0$	B1	
	Correctly applies quadratic formula: $[x =] \frac{-(-3y) \pm \sqrt{(-3y)^2 - 4(2)(-1)}}{2(2)} \text{ oe}$ or $[y =] \frac{-(-3x) \pm \sqrt{(-3x)^2 - 4(2)(-1)}}{2(2)} \text{ oe}$	M1	FT their $2x^2 - 3xy - 1 = 0$ or $2y^2 - 3xy - 1 = 0$ with at most one sign error in the equation
	Justifies the positive square root at some point	B1	
	$f^{-1}(x) = \frac{3x + \sqrt{9x^2 + 8}}{4} \text{ cao}$	A1	must be a function of x

-	1		
9(a)	f > 3	B1	
9(b)	Complete method: Putting $y = f(x)$ and changing subject to x and swapping x and y or swopping x and y and changing subject to y	M1	must be a function of x not y
	$f^{-1}(x) = \frac{1}{5}\ln(x-3)$	A1	
	[Domain] $x > 3$	B1	FT their part (a) providing their part (a) is of the form $f > a$ or $f \ge a$, where a is a constant
9(c)	their $\ln(x-3) = \ln 1$ soi or their $(x-3) = e^0$ soi or $f(0) = 3 + e^{5\times 0}$ soi	M1	FT their $f^{-1}(x)$ of the form $k \ln(\pm x \pm 3)$, where k is a non-zero constant
	x = 4	A1	
9(d)	Fully correct graph: $y = f(x)$ $y = f^{-1}(x)$	B4	B2 for correct exponential shape for f, in 1st and 2nd quadrants, with correct asymptotic behaviour soi or B1 for a correct exponential shape for f, in 1st and 2nd quadrants, with asymptotic behaviour but to a clearly incorrect line $y = k$ soi B1 for f ⁻¹ reflection of f in the line $y = x$ B1 for intercepts (0, 4) and (4, 0) and no others; must have attempted correct exponential and logarithmic shapes Maximum of 3 marks if not fully correct

1(a)	$-3 < x < 1 \qquad x > 5$	B1	
1(b)	$-\frac{1}{3}(x+3)(x-1)(x-5)$	3	B1 for a negative cubic function B1 for a cubic function multiplied by $\frac{1}{3}$ B1 for $(x+3)(x-1)(x-5)$

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1(a)	$y = -\frac{1}{2}(x+2)(x+1)(x-5)$	B1	For $-\frac{1}{2}$
		B1	For $(x+2)(x+1)(x-5)$
1(b)	$-2 \leqslant x \leqslant -1$	B1	
	x ≥ 5	B1	

5(a)	f>9	B1	Allow y but not x
5(b)	It is a one-one function because of the restricted domain	B1	
5(c)	$x = (2y + 3)^2$ or equivalent	M1	For a correct attempt to find the inverse
	$y = \frac{\sqrt{x} - 3}{2}$	M1	For correct rearrangement
	$f^{-1} = \frac{\sqrt{x} - 3}{2}$	A1	Must have correct notation
5(d)	x > 9	B1	FT on their (a)
5(e)	$f(\ln(x+4)) = 49$	M1	For correct order
	$(2\ln(x+4)+3)^2 = 49$ $\ln(x+4) = 2$	M1	For correct attempt to solve, dep on previous M mark, as far as $x =$
	$x = e^2 - 4$	A1	

. 11(a)	y de la constant de l	В3	B1 for correct shape of f or f ¹ B1 for symmetry B1 for drawn over correct domain Maximum of 2 marks if not fully correct
11(b)(i)	$[\pm]\sqrt{x-1} = y-4 \text{ soi}$	M1	
	$g^{-1}(x) = 4 - \sqrt{x-1}$	A1	
	[Range] $g^{-1} \leq 4$	B1	
	[Domain] $x \ge 1$	B1	
11(b)(ii)	$\ln(2[(x-4)^2+1]+1)$	M1	
	$\ln(2x^2 - 16x + 35)$	A1	
11(b)(iii)	Valid explanation, e.g. some of the values in the range of f are outside the domain of g	B1	

-	2(a)	$y = -\frac{1}{2}(x+5)(x+1)(x-2)$	3	B1 for negative soi B1 for $\frac{1}{2}$ soi B1 for $(x+5)(x+1)(x-2)$ or $x^3 + 4x^2 - 7x - 10$
	2(b)	-5 < x < -1	B1	
		x > 2	B1	

6(a)	It is a one-one function because of the given restricted domain or because $x \ge -1$	B 1	
6(b)	7 p=P(r)	4	B1 for $y = f(x)$ for $x > -1$ only B1 for 1 on x-axis and -3 on y-axis for $y = f(x)$ B1 for $y = f^{-1}(x)$ as a reflection of $y = f(x)$ in the line $y = x$, maybe implied by intercepts with axes B1 for 1 on y-axis and -3 on x-axis for $y = f^{-1}(x)$

3(a)(i)	$x > \frac{1}{2}$	B1	Must be using <i>x</i>
3(a)(ii)	$x = 4\ln(2y - 1)$ $e^{\frac{x}{4}} = 2y - 1$ $y = \frac{1}{2}\left(1 + e^{\frac{x}{4}}\right)$	M1	For full method for inverse using correct order of operations
	$f^{-1}(x) = \frac{1}{2} \left(1 + e^{\frac{x}{4}} \right) \text{ or } f^{-1}(x) = \frac{1}{2} \left(1 + \sqrt[4]{e^x} \right)$	A1	Must be using correct notation
	$x \in \mathbb{R}$	B1	
3(b)	$\sqrt{2x-3}+5=7$	M1	For correct order
	$x = \frac{2^2 + 3}{2}$	M1	Dep on previous M mark, for obtaining x by simplifying and solving using correct order of operations, including squaring
	$x = \frac{7}{2}$ or 3.5	A1	

8(i)	f > -1	B1	or $f(x) > -1$, $y > -1$, $(-1, \infty)$, $\{y : y > -1\}$
8(ii)	$e^y = \frac{x+1}{5} \text{ oe}$	M1	a complete valid method to obtain the inverse function
	$y = \ln\left(\frac{x+1}{5}\right)$ or $f^{-1}(x) = \ln\left(\frac{x+1}{5}\right)$ oe	A1	
	Domain $x > -1$ or $(-1, \infty)$	B1	FT their (i) or correct
8(iii)	g(1) = 5 so fg(1) = f(5)	M1	evaluation using correct order of operations
	$5e^5 - 1 = 741$	A1	awrt 741 or 5e ⁵ –1
8(iv)	$g^{2}(x) = (x^{2} + 4)^{2} + 4$	M1	correct use of g ²
	$x^{4} + 8x^{2} + 16 + 4 = 40$ $(x^{2} + 4)^{2} = 36$ or $x^{4} + 8x^{2} - 20 = 0$ $(x^{2} + 10)(x^{2} - 2) = 0$	M1	DepM1 for forming and solving a quadratic in x^2
	$x = \pm \sqrt{2}$ only	A1	

 		+	
12(a)(i)	g > -9	B1	
12(a)(ii)	x > 1	B1	
12(a)(iii)	$[gf(x) =] 4(5x-2)^2-9$	B1	
	$100x^2 - 80x - 38 = 0$	M1	
	or $(5x-2)^2 = \frac{45+9}{4}$		
	$[x =] \frac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$		
	leading to $\frac{4+3\sqrt{6}}{10}$ oe only	A1	
	or $\frac{1}{5} \left(2 + \sqrt{\frac{54}{4}} \right)$ or better only		
12(b)(i)	(They are) reflection s (of each other) in (the line) $y = x$ oe	B1	
12(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	M1	
	$x = [\pm]\sqrt{y^2 + 1}$ or $y = [\pm]\sqrt{x^2 + 1}$	A1	
	$-\sqrt{x^2+1}$ nfww	A1	

5a(i)	fg	B1	
5a(ii)	g ⁻¹	B1	
5a(iii)	f^{-1}	B1	
5a(iv)	g^2	B1	
5(b)(i)	Undefined at $x = 0$ oe	B1	
5(b)(ii)	$4 = a + b$ $h'(x) = \frac{p}{x^3} \text{ and attempt at } h'(1)$	M1	For attempt at h(1) and differentiation to obtain h'(1), must have the form $h'(x) = \frac{p}{x^3}$ oe
	b = -8 $a = 12$	A1	For both

	i	1	I
10(i)	$gf(x) = e^{2(\ln(3x+2))} - 4$	B1	
	their gf = 5	M1	
	use $\ln a^p = p \ln a$ or $e^{\ln a} = a$ or $\ln e^a = a$	B1	correct use of log/exponential relationship seen anywhere
	$3x + 2 = 3 \text{ or } (3x + 2)^2 = 9$	A1	3 may take the form of e ^{0.5ln9} 9 may take the form of e ^{ln9}
	$x = \frac{1}{3}$ only	A1	
10(ii)	$x = \frac{e^y - 2}{3}$	M1	find x in terms of y
	$\frac{e^x - 2}{3} \left(= f^{-1}(x) \text{ or } = y\right)$	A1	interchange x and y correct completion
10(iii)	$\frac{e^x - 2}{3} = e^{2x} - 4$	M1	their $f^{-1}(x) = g(x)$
	$3e^{2x} - e^x - 10 \ (=0)$	A1	obtain quadratic in e ^x must be arranged as a three term quadratic in order shown
	$(3e^x + 5)(e^x - 2) (= 0)$	M1	solve for e ^x
	$x = \ln 2$ or 0.693 only	A1	

18.

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A	В	С	D
	✓		
		✓	✓
		✓	
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B1 for either each row correct or each column correct – mark to candidate's advantage.

. 1	9(i)	$5\left(x-\frac{7}{5}\right)^2-\frac{64}{5}$	В3	B1 for each of p , q , r correct in correct format; allow correct equivalent values. If B0 , then SC2 for $5\left(x-\frac{7}{5}\right)-\frac{64}{5}$ or SC1 for correct values but incorrect format
	9(ii)	-0.2 0 3	B4	B2 for fully correct shape in correct position or B1 for fully correct shape translated parallel to the <i>x</i> -axis B1 for <i>y</i> -intercept at (0, 3) marked on graph B1 for roots marked on graph at -0.2 and 3
•	9(iii)	$0 < k < \left their \left(-\frac{64}{5} \right) \right $	B2	FT their (i) B1 for any inequality using their $\frac{64}{5}$ or max y value is their 12.8soi

10(a)(i	-3	B2	B1 for correct shape B1 for roots marked on the graph or seen nearby provided graph drawn and one root is negative and one is positive
10(a)(i	Any correct domain	B1	
10(b)(i	$\frac{4}{3x-1}$	B1	mark final answer

10(b)(ii)	Correct method for finding inverse function e.g. swopping variables and changing subject or vice versa; or indicates $(hg)^{-1}(x) = g^{-1}h^{-1}(x)$ and finds $g^{-1}(x) = \frac{x+1}{3}$ and $h^{-1}(x) = \frac{4}{x}$	M1	FT only if $their$ hg(x) of the form $\frac{a}{bx+c}$ where a , b and c are integers
	$\left[(hg)^{-1}(x) = \right] \frac{1}{3} \left(\frac{4}{x} + 1 \right) \text{ oe isw or}$ $\left[(hg)^{-1}(x) = \right] \frac{4+x}{3x} \text{ oe isw}$	A1	FT their (hg) ⁻¹ (x) = $\frac{a - cx}{bx}$ oe If M0 then SC1 for their hg(x) of the form $y = \frac{a}{x} + b \text{ oe leading to their (hg)}^{-1}(x) \text{ of the}$ form $y = \frac{a}{x - b}$ isw
10(c)	a cao	B1	

10(a)(ii)	Any correct domain	B1	
10(a)(ii)	Any correct domain	B1	
10(b)(i)	$\frac{4}{3x-1}$	B1	mark final answer
10(b)(ii)	Correct method for finding inverse function e.g. swopping variables and changing subject or vice versa; or indicates $(hg)^{-1}(x) = g^{-1}h^{-1}(x)$ and finds $g^{-1}(x) = \frac{x+1}{3}$ and $h^{-1}(x) = \frac{4}{x}$	M1	FT only if $their$ hg(x) of the form $\frac{a}{bx+c}$ where a , b and c are integers
	$\left[(hg)^{-1}(x) = \right] \frac{1}{3} \left(\frac{4}{x} + 1 \right) \text{ oe isw or}$ $\left[(hg)^{-1}(x) = \right] \frac{4+x}{3x} \text{ oe isw}$	A1	FT their (hg) ⁻¹ (x) = $\frac{a - cx}{bx}$ oe If M0 then SC1 for their hg(x) of the form $y = \frac{a}{x} + b \text{ oe leading to their (hg)}^{-1}(x) \text{ of the}$ form $y = \frac{a}{x - b}$ isw
10(c)	a cao	B1	

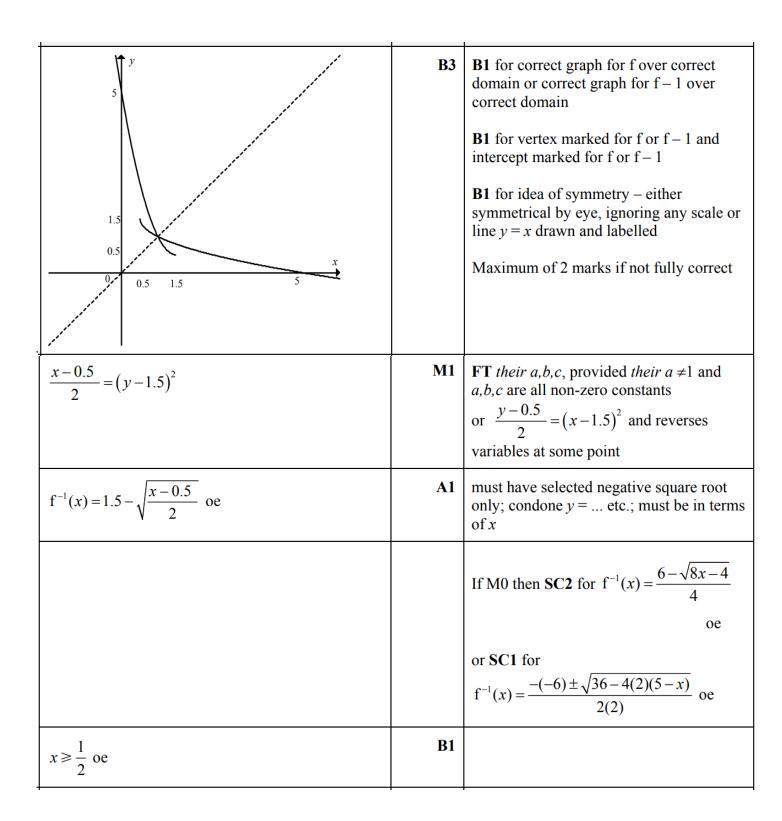
11(i)	$gf(x) = \frac{2(4x-3)+1}{3(4x-3)-1}$	M1	
	$= \frac{8x - 5}{12x - 10}$	A1	
11(ii)	y(3x-1) = 2x+1 or $x(3y-1) = 2y+1$	B1	
	(3y-2)x = y+1 or $(3x-2)y = x+1$	M1	
	$g^{-1}\left(x\right) = \frac{x+1}{3x-2}$	A1	
11(iii)	$4\left(\frac{2x+1}{3x-1}\right) - 3\left[=x-1\right]$	B1	
	$3x^2 - 3x - 6$ oe	B1	
	3(x+1)(x-2)	M1	
	x = 2 only	A1	

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8(i)	$5 \le f(x) \le 6$ or $[5,6]$ oe	B2	B1 for $5 \leqslant f(x) \leqslant p \ (p > 5)$ or for $q \leqslant f(x) \leqslant 6 \ (q < 6)$
8(ii)	$x = \sin\frac{y}{4} + 5$	M1	complete valid attempt to obtain the inverse with operations in correct order.
	$y = 4\sin^{-1}\left(x - 5\right)$	A1	
	Range $0 \leqslant y \leqslant 2\pi$	B1	
8(iii)	$2\left(\sin\frac{\left(x-\frac{\pi}{3}\right)}{4}+5\right) (=11)$	B1	for $\sin \frac{\left(x - \frac{\pi}{3}\right)}{4} + 5$
	$\sin\frac{\left(x-\frac{\pi}{3}\right)}{4} = \frac{1}{2}$	M1	for $\sin \frac{\left(x - \frac{\pi}{3}\right)}{4} = k$
	$x = 4\sin^{-1}\left(\frac{1}{2}\right) + \frac{\pi}{3} \text{ oe}$	M1	Dep for use of \sin^{-1} and correct order of operations to obtain x. Allow one +/- or ×/ ÷ sign error
	$x = \pi \text{ or } 3.14$	A1	$x = \pi$ and no other solutions in range

4(a)(i)	f > 5, f(x) > 5	B1	
4(a)(ii)	$\frac{y-5}{3} = e^{-4x}$ or $\frac{x-5}{3} = e^{-4y}$	B1	
	$-4x = \ln\left(\frac{y-5}{3}\right) \text{ or } -4y = \ln\left(\frac{x-5}{3}\right)$	В1	
	leading to $f^{-1}(x) = -\frac{1}{4}\ln\left(\frac{x-5}{3}\right)$ or $f^{-1}(x) = \frac{1}{4}\ln\left(\frac{3}{x-5}\right)$ or $f^{-1}(x) = \frac{1}{4}(\ln 3 - \ln(x-5))$ or $f^{-1}(x) = -\frac{1}{4}(\ln(x-5) - \ln 3)$	В1	
	Domain $x > 5$	B1	
4(b)	$\ln\left(x^2 + 5\right) = 2$	B1	
	$x^2 + 5 = e^2$	B1	
	$x = 1.55$ or better or $\sqrt{e^2 - 5}$	B1	

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).	$2(x-1.5)^2 + 0.5$ isw	В3	or B3 for $a = 2$ and $b = 1.5$ and $c = 0.5$ provided not from wrong format isw
			or B2 for $2(x-1.5)^2 + c$ where $c \neq 0.5$ or $a = 2$ and $b = 1.5$
			or SC2 for $2(x-1.5)+0.5$ or $2((x-1.5)^2 + \frac{1}{4})$ seen
			$2\left(\left(x-1.5\right)^2+\frac{1}{4}\right) \text{ seen}$
			or B1 for $(x-1.5)^2$ seen or for $b = 1.5$ or for $c = 0.5$
			or SC1 for 3 correct values seen in
			incorrect format e.g. $2(x-1.5x)+0.5$ or
			$2(x^2-1.5)+0.5$



6(i)	$y \in \mathbb{R}$ oe	B1	Must have correct notation i.e. no use of x
6(ii)	y > 3 oe	B1	Must have correct notation i.e. no use of x
6(iii)	$f^{-1}(x) = e^x \text{ or } g(4) = 35$	B1	First B1 may be implied by correct answer or by use of 35
	$f^{-1}g(4) = e^{35}$	B 1	
6(iv)	$\frac{y-3}{2} = x^2 \text{ or } \frac{x-3}{2} = y^2$	M1	valid attempt to obtain the inverse
	$g^{-1}(x) = \sqrt{\frac{x-3}{2}}$	A1	correct form, must be $g^{-1}(x) = or$ y =
	Domain $x > 3$	B1	Must have correct notation

6(i)	$f^2 = f(f)$ used algebraic $([(x+2)^2 + 1] + 2)^2 + 1$	M1	numerical or algebraic
	17	A1	
6(ii)	$x = \frac{y-2}{2y-1}$	M1	change x and y
	$2xy - x = y - 2 \rightarrow y(2x - 1) = x - 2$	M1	M1dep multiply, collect <i>y</i> terms, factorise
	$y = \frac{x-2}{2x-1} \qquad \left[= g(x) \right]$	A1	correct completion
6(iii)	gf(x) = $\frac{[(x+2)^2+1]-2}{2[(x+2)^2+1]-1}$ oe	B1	
	$\frac{(x+2)^2 - 1}{2(x+2)^2 + 1} = \frac{8}{19}$ $3(x+2)^2 = 27 \text{ oe } 3x^2 + 12x - 15 = 0$	M1	their gf = $\frac{8}{19}$ and simplify to quadratic equation
	solve quadratic	M1	M1dep Must be of equivalent form
	x=1 $x=-5$	A1	